Appendices

Appendix 1: Solving problem

A1 - §1: The force acting by the water in the control volume on the pipe walls F is the same as the force acting by the pipe on the water in the control volume F_b, but in the opposite sense. From here, Equation 1 can be modified into a special form to accommodate the solution of this problem:

$$-\vec{F}_b = -\int_{S_c} \vec{V}\,\mathrm{d}\dot{m} + \vec{F}_h + \vec{F}_p = \vec{F}.$$

The entered parameters are:

d	z	z_{H2O}	V
23	1,2	2	4

d [mm]; z [m]; V [m·s^{-1}]

A1 - §2: The integration of the equation from the first paragraph is easy because the output and input to the control volume is only at locations 1 and 2 of the mean velocity V. The flow direction at location 1 is the same as the z-axis direction (but against the normal of the control volume surface) and at location 2 is the same as the x-axis direction:

$$-\int_{S_c} \vec{V}\,\mathrm{d}\dot{m} = -V\cdot\dot{m}\,\vec{\mathrm{i}} + V\cdot\dot{m}\vec{\mathrm{k}}.$$

The individual components of the force F acting on the pipe can therefore be calculated according to the following equations:

$$F_x = -V\cdot\dot{m} + F_{h,x} + F_{p,x},\; F_y = V\cdot\dot{m} + F_{h,y} + F_{p,y},\; F_z = F_{h,z} + F_{p,z}.$$

A1 - §3: The calculation of the force component F_x can be based on the diameter of the pipe and the density of the water, which is not specified, so the standard value is used.

The fluid is subject only to the external gravitational acceleration g, which acts in the opposite direction of the z-axis.

For the atmospheric pressure, substitute the standard value:

ρ	g	p_{at}
1000	9,81	101,33

ρ [kg·m^{-3}]; g [m·s^{-2}]; p [kPa]

A1 - §4: The pressure in the pipe $A2$ at flange 2 is applied to the control volume in direction-x over an area corresponding to the pipe diameter. The pressure $A2$ can be calculated using Bernoulli's equation from the pressure $A1$ and the height z. The pressure $A1$ can be calculated from the pressure pat and the height z_{H2O}.

$F_{h,x}$	A	m	$A1$	$A2$	$F_{p,x}$	F_x
0	415,48	1,6619	120,95	109,17	-45,359	-52,006

F [N]; A [mm^2]; m [kg·s^{-1}]; p [kPa]

A1 - §5: The weight $F_{h,z}$ in the direction of the z-axis corresponds to the product of the gravitational acceleration and the weight of the water enclosed in the control volume, The volume of water in the pipe elbow corresponds to approximately half the volume of a cylinder of pipe diameter *d*:

V_C	m	$F_{h,z}$	$F_{p,z}$	F_z
0,5033	0,5033	-4,9379	50,250	51,960

V_C [l]; m [kg]; F [N]

A1 - §6: If the effect of the Coriolis acceleration from the rotation of the earth, which causes an additional external force in the control volume, is ignored in the calculation, then the force in the y-direction will be zero.

Appendix 2: Solving problem

A2 - §1: The calculation is performed according to Equation 2 applied to the control volume V_C, whose boundary passes through the center of the blade passages and along the inner and outer radii, see Fig. The calculation neglects the effect of the gravitational forces F_h because the orientation of the impeller with respect to the direction of gravitational acceleration is not defined and, in addition, the air has a very low density:

$$\vec{F}=\vec{V}_1\frac{\dot{m}}{Z}-\vec{V}_2\frac{\dot{m}}{Z}+\vec{F}_{p},$$

index $_R$ indicates that this is force acting on all rotor blades.

The blades are not able to absorb the axial component of the force, which is evident from their orientation, and even the absolute velocity V has no components in the axial direction. Therefore, the force F has only radial and tangential components:

$$F_r=V_{1r}\frac{\dot{m}}{Z}-V_{2r}\frac{\dot{m}}{Z}+F_{p,r}$$

$$F_\theta=V_{1\theta}\frac{\dot{m}}{Z}-V_{2\theta}\frac{\dot{m}}{Z}+F_{p,\theta}.$$

The entered parameters are:

m	$A1$	Z	r_1	r_2	V_1	V_2	α_2	b
88,8	101,33	52	32,5	37,5	3,4	9,34	18,4	30

m [kg·h^{-1}]; p [kPa]; Z [-]; r, b [mm] impeller width (blade length); V [m·s^{-1}]; α [°]

A2 - §2: The resultant of the pressure forces $F_{p,r}$ will be non-zero even if the pressures $A1$ and $A2$ are equal. The area between the points AB is larger than the area CD, so we can write:

$$F_{p,r}=-\frac{p_1 2\pi r_2\cdot b-p_1 2\pi r_1\cdot b}{Z}=p_1 2\pi b\frac{r_1-r_2}{Z}.$$

V_{2r}	$F_{r,p}$	F_r
2,948	-1,836	-1,836

V [m·s^{-1}]; F [N]

A2 - §3: The pressure forces in the tangential direction at the BC and DA boundaries are cancelled: