

INTRODUCTION TO TURBOMACHINERY

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Defining features of turbomachines

Rotor (Impeller)
Blade
Blade passage
Kaplan turbine

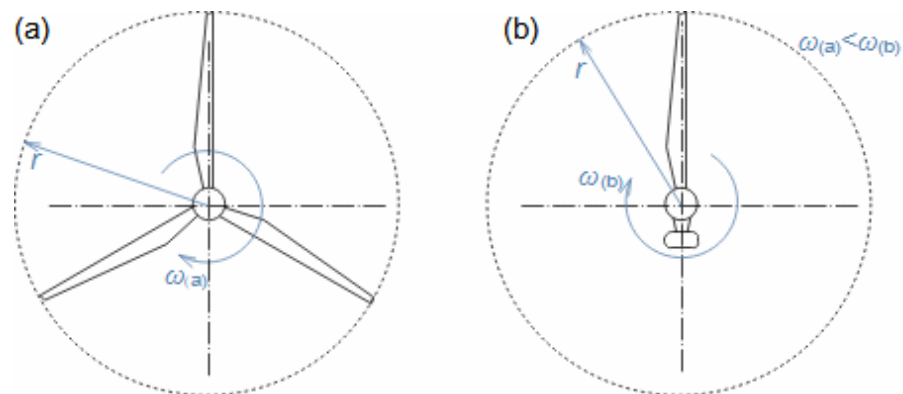
Blade machines are a wide group of machines (e.g. steam turbines, gas/combustion turbines, turbochargers, centrifugal/radial pumps, water turbines, etc.). Their characteristic feature is the rotor, which is a shaft with blades around its circumference (called an impeller). The blades form a so-called blade passage in which the working fluid flows - [Figure 1](#) shows the impeller of a Kaplan water turbine, in which the blade passages are clearly visible. The energy transformation occurs due to the interaction of forces between the working fluid and the blades.



1: Kaplan turbine – rotor

Blade passage
Wind turbine
Rotor speed

The blade channel principle also works for "sparse" rotors, or even with large blade spacing, as demonstrated by [wind turbine](#)¹⁰: rotors, see [Figure 2](#). Even single-blade rotors can be constructed. In general, however, the smaller the number of blades, the higher the rotor speed for the same change of direction of velocity in the blade passage as a rotor with a larger number of blades but lower speed - only in this way can the flow over the entire rotor area be processed with the same efficiency.



2: Wind turbine - rotor

ω [$\text{rad}\cdot\text{s}^{-1}$] angular velocity; r [m] rotor radius.

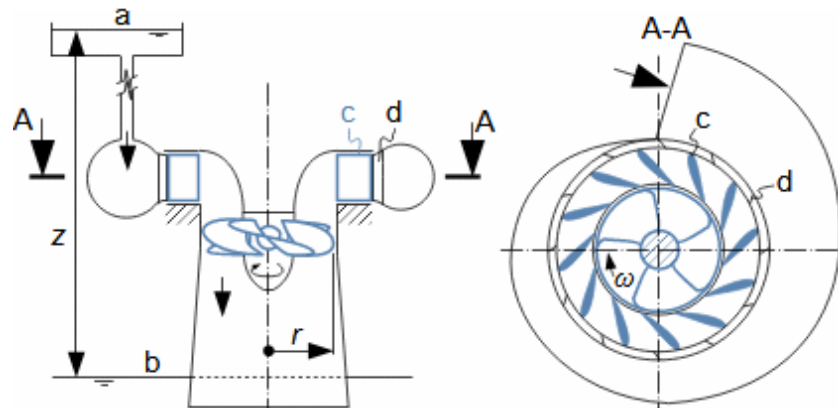
Operating principle of turbomachines

Turbine vs. Working machine

The rotation of the rotor of turbomachine is caused by the forces acting on the blades. If the working fluid imparts energy to the rotor, then the machine is called a turbine (the action force is from the flow of working fluid reacting from the blades) - the machine is doing work. In pumps, turbochargers, fans - working machines for short - the opposite process takes place and the working fluid gains energy (the action force is from the blades reacting with the fluid flow) - the machine consumes the work. These forces are generated during energy transformations inside the blade passages, and in turbomachines, depending on the type, pressure, kinetic, potential or internal thermal energy can be transformed.

Kaplan turbine
Stator (Quide vanes)

For turbomachines, a pressure difference upstream and downstream of the machine (pressure gradient) or a difference in the velocity of the working fluid or a combination of both is typical, as for example for a water Kaplan turbine, see Figure 3. This water turbine also contains blades outside the impeller, such blades are referred to as stator blades and their purpose is to direct the working fluid flow at the required angle and velocity towards the rotor row of blades. The stator row of blades also transforms part of the pressure energy of the water column above the turbine into kinetic energy. The stator (stator blades) is contained in most types of turbomachinery.



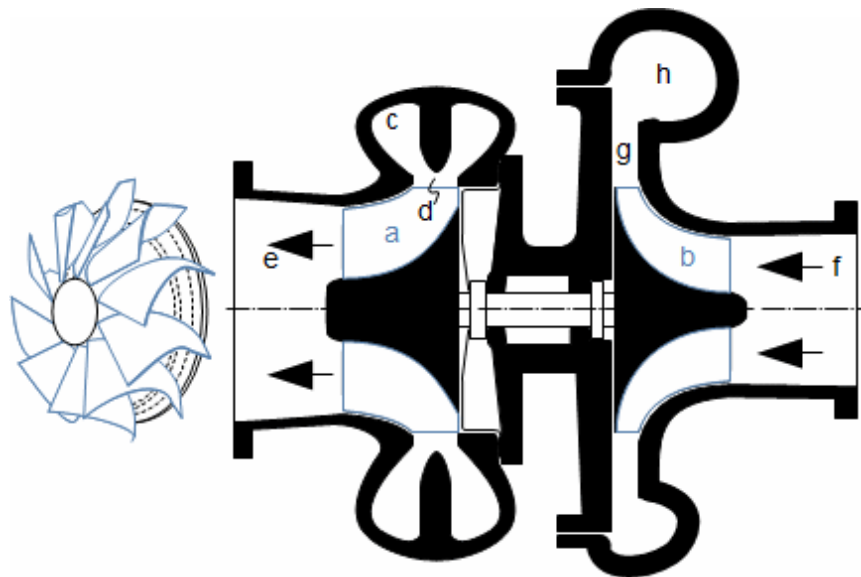
3: Energy transformation in Kaplan turbine

a-inlet reservoir level; b-outlet reservoir level; c-guide vanes (stator); d-reinforcement of spiral casing (sometimes called volute). z [m] height difference between levels.

Turbocharger

Vaneless stator
(Vaneless confuser,
Vaneless diffuser)

An internal combustion engine turbocharger is a turbomachine with two rotors on a common shaft, see [Figure 4](#) - one rotor is a turbine rotor and drives the compressor rotor, which compresses air for the engine. In this case, the exhaust gases from the engine enter the turbine rotor through two spiral casings which discharge into a vaneless stator which performs the same function as the blade stator in a Kaplan turbine. In the compressor rotor, the intake air is compressed and at the same time accelerated. At the outlet of the compressor impeller is a vaneless stator (diffuser) whose function is to move the air evenly away from the impeller and slow it down before entering the spiral casing.

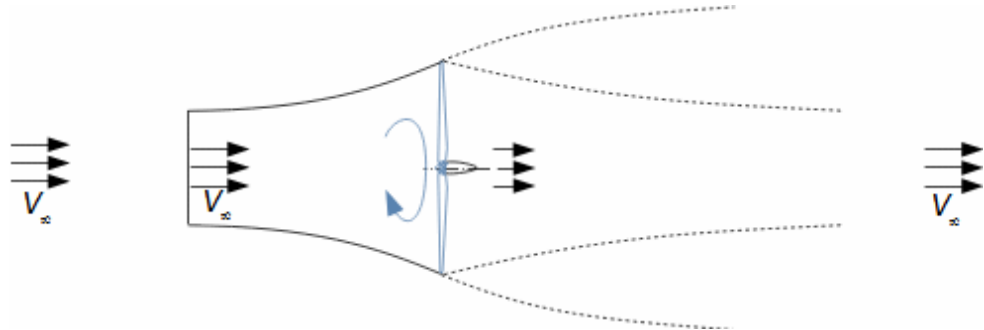


4: Turbocharger

a-turbine impeller; b-compressor impeller; c-double spiral casing of turbine; d-vaneless confuser; e-exhaust; f-compressor inlet; g-vaneless diffuser; h-spiral casing of compressor.

Wind turbine

Wind turbines¹¹ have the largest rotors, see Figure 5. In this case, the kinetic energy of wind is transformed into work. Wind turbines do not have a casing, so the flow behind the turbine is influenced by the surrounding flow with higher kinetic energy.



5: Wind turbine

V_∞ [$\text{m} \cdot \text{s}^{-1}$] wind velocity in front of affected turbine area.

Basic types of turbomachines

Hydraulic machines

Heat machines

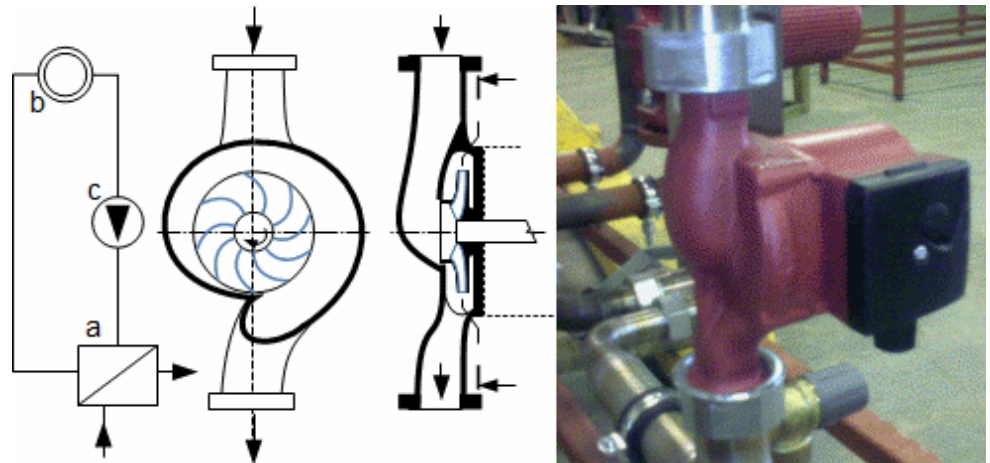
The design process of turbomachine is mostly influenced by the properties of the working fluid, more precisely its compressibility, which is why we divide turbomachines into hydraulic and thermal machines. For hydraulic machines (pumps, fans, etc.) the change in the density of the working fluid is largely insignificant. For heat machines (compressors, steam turbines, etc.) the density of the working fluid changes significantly.

Rotodynamic pumps

Pumps operating on the principle of the turbomachine are called turbopumps or hydrodynamic pumps⁷. Pumps are machines used to transport and pressurize a fluid. Rotodynamic pumps can be divided into circulating (circulating), condensate and feed pumps according to the operating conditions.

Circulation pump

Circulation pumps are mainly used to circulate the liquid in the loop - overcoming pressure losses in the loop. The energy transferred to the fluid in a circulator is approximately $100 \text{ J} \cdot \text{kg}^{-1}$. The power input can be up to units of MW (the main circulation pump of a nuclear power plant). Figure 6 shows an example of a small circulator with a centrifugal or radial impeller in a monoblock design, which is connected in a loop with a heat exchanger and a heat consumer. The fluid in the impeller, by centrifugal forces, flows from the centre of the impeller to its circumference. From the rotor, the fluid exits into a spiral casing where it is discharged to the discharge end of the pump.



6: Circulation pump

a-heat exchanger; b-heat consumer; c-circulation pump.

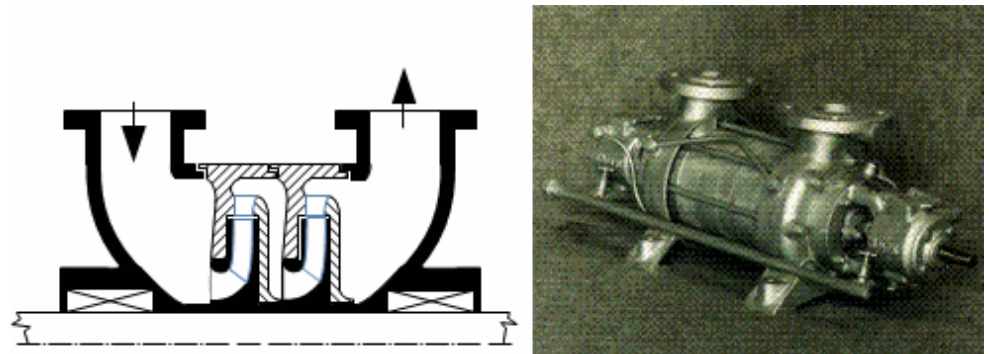
Condensate pump

Condensate pumps are designed to pump liquids close to the saturation point (e.g. condensate and liquefied gases). The energy transferred to the liquid in a condensate pump is higher than in circulation pumps ($500 \text{ J}\cdot\text{kg}^{-1}$ in the case of water) because the condensate is pumped to higher pressures.

Feed pump

Multi-stage
turbomachine

Feed pumps are characterized by pumping the liquid to high pressures, where the energy transferred to the liquid reaches up to several tens of $\text{kJ}\cdot\text{kg}^{-1}$ - in order to transfer this amount of energy to the liquid, several impellers are required in succession, in such cases we speak of a multi-stage turbomachine, see [Figure 7](#).



7: Multi-stage pump

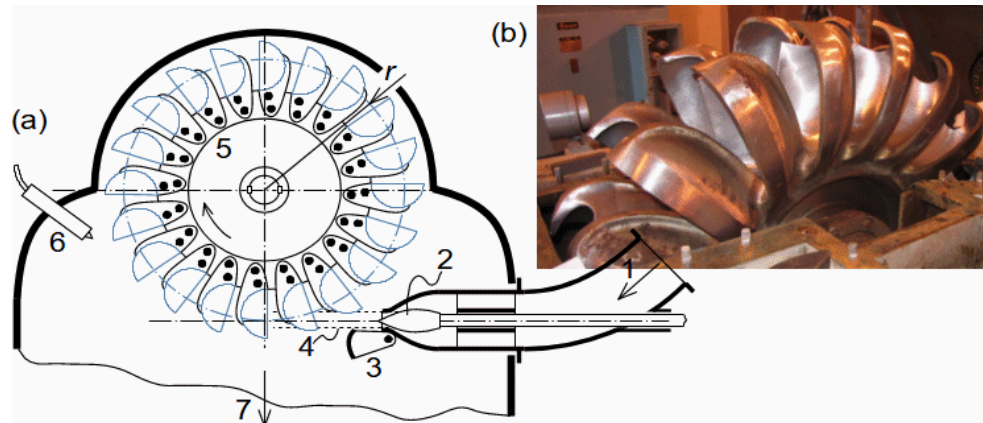
Pictured is a pump from Sigma Hranice [Anon., 2009b].

Water turbines

Water turbines are among the most powerful types of turbomachinery with outputs up to 1000 MW (see also [Škorpík, 2020]). Three types of water turbines are most commonly used: the Pelton turbine, the Francis turbine, and the Kaplan turbine. A water turbine needs a specific minimum difference in levels or pressures.

Pelton turbine

In Pelton turbines, the potential energy of water is first transformed into kinetic energy in the nozzle in front of the impeller. The jet of water from the nozzle spins the impeller as it contacts its blades, where it transfers its kinetic energy to them, see [Figure 8](#).

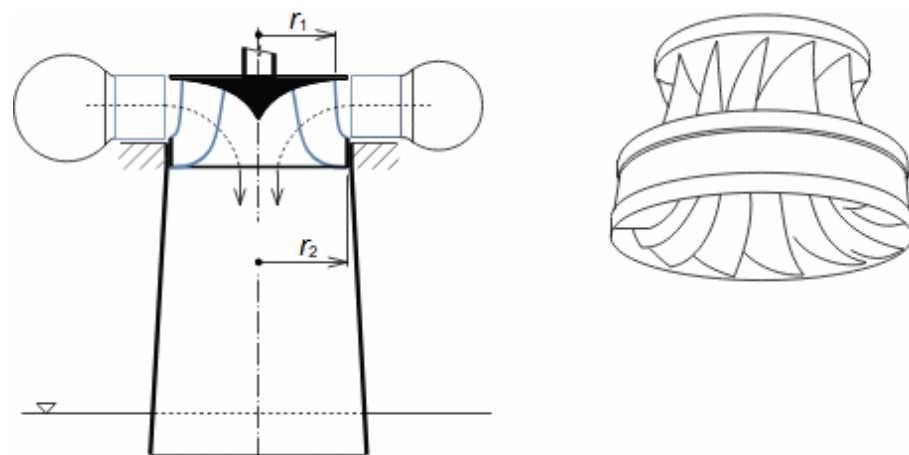


8: Pelton turbine

(a) basic parts of Pelton turbine; (b) rotor of Pelton turbine with diameter 850 mm and power 980 kW - this turbine is part of Temelín nuclear power plant, from which waste water is fed through pipe 6.47 km long and 700 mm in diameter, turbine's machine room is at level of Vltava river at Kořensko hydroelectric power plant, author of photograph is Jiří Kohout. 1-inlet from ball valve; 2-regulating needle; 3-deviator (deviator) of water jet; 4-water jet; 5-blades; 6-braking nozzle (reduces turbine run time during shutdown); 7-water outlet through shaft.

Francis turbine
Kaplan turbine

Francis turbines is similar to Kaplan turbines. There is pressure corresponding to the water head in front of the stator row of blades. In the stator, the water flow accelerates (due to the narrowing of the channels created by the stator blades) and the pressure drops. The water stream enters the rotor blade passages, which spin the rotor. The stator blades are rotatable, which allows power control, see [Figure 9](#). The Kaplan turbine, unlike the Francis turbine, also has rotatable rotor blades, see [Figure 1](#).



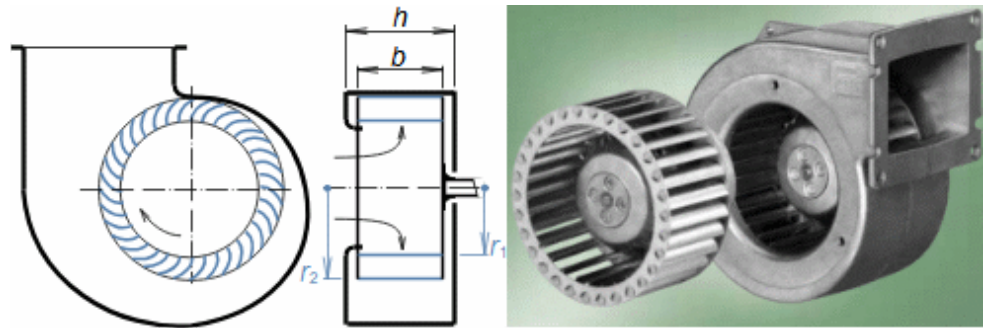
9: Francis turbine

Fans

Fans⁸ are used for transport and for a small increase in gas pressure at which there are no significant changes in density. According to the increase in total pressure, fans are divided into low pressure (0 to 1 kPa), middle pressure (up to 3 kPa) and high pressure (above 3 kPa).

Centrifugal low pressure fan

Figure 10 shows a section of a low pressure radial fan with forward curved blades with a spiral casing. In this case, only the working gas velocity is increased in the impeller, since the blade passages have a constant flow area, the working gas pressure can be increased in the diffuser passage connected after the outlet of the spiral casing.



10: Radial low pressure fan

b [m] width of impeller; h [m] width of spiral casing. Photo: ebmpapst [Anon., 2009a], casing cast in aluminium alloy.

Wind turbines
Vortex machine
Propeller
Stream tube

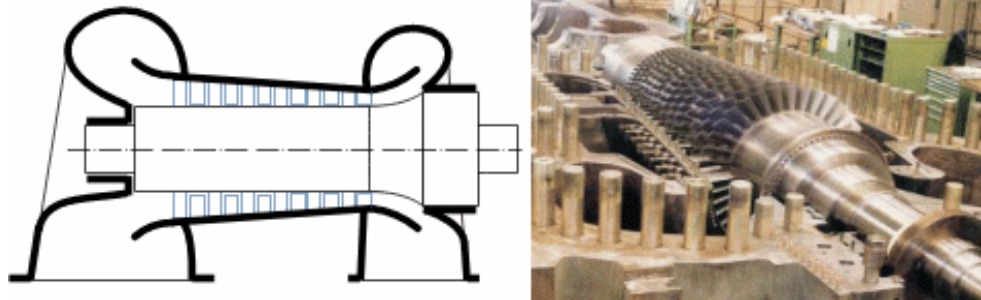
Machines without a casing containing only a rotor are also called vortex machines because there must be a vortex behind the rotor. Machines without a casing include wind turbines (Figure 11), aircraft propellers or ship propellers. Machines without a casing can only handle small changes in pressure because this would lead to instability of the rotor stream tube¹⁰, see Figure 5.



11: The rotor of the Vestas V90 wind turbine with a column height of 105 m, a rotor diameter of 90 m and an installed capacity of 2 MW. Drahaný (CZ).

Turbocompressors

In turbocompressors, the compression of gases or vapours takes place, or the increase in pressure energy and, if the compression is not cooled, the increase in internal thermal energy due to the increase in temperature. The turbopressor blade passages are in the form of diffusers in which the kinetic energy of the gas is transformed into enthalpy. For higher compression, multi-stage turbocompressors are used, see [Figure 12](#).



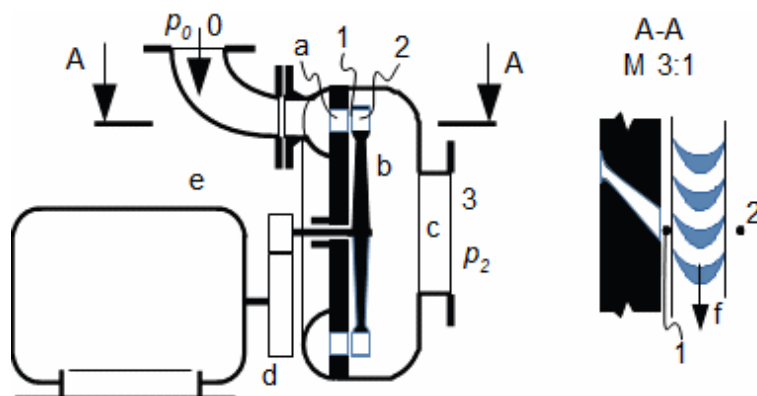
12: Multi-stage turbocompressor
Photos from [Anon., 2009d].

Steam turbines

In steam turbines, vapour (usually water) expands to a lower pressure while its enthalpy is transformed into work. Steam turbines are used to generate electricity in thermal and nuclear power plants and in industrial plants with a steam source.

Laval turbine
Nozzle

[Figure 13](#) shows a section of a Laval steam turbine, in order to describe its function. Steam from state 0 expands to state 1 in a stator in the form of the Laval nozzle, in which enthalpy is transformed into kinetic energy [Škorpík, 2023b]. The steam stream then enters the rotor blade passages in which the kinetic energy of the steam is transformed into work. The work done is therefore the difference of the kinetic energies in front of and behind the rotor.

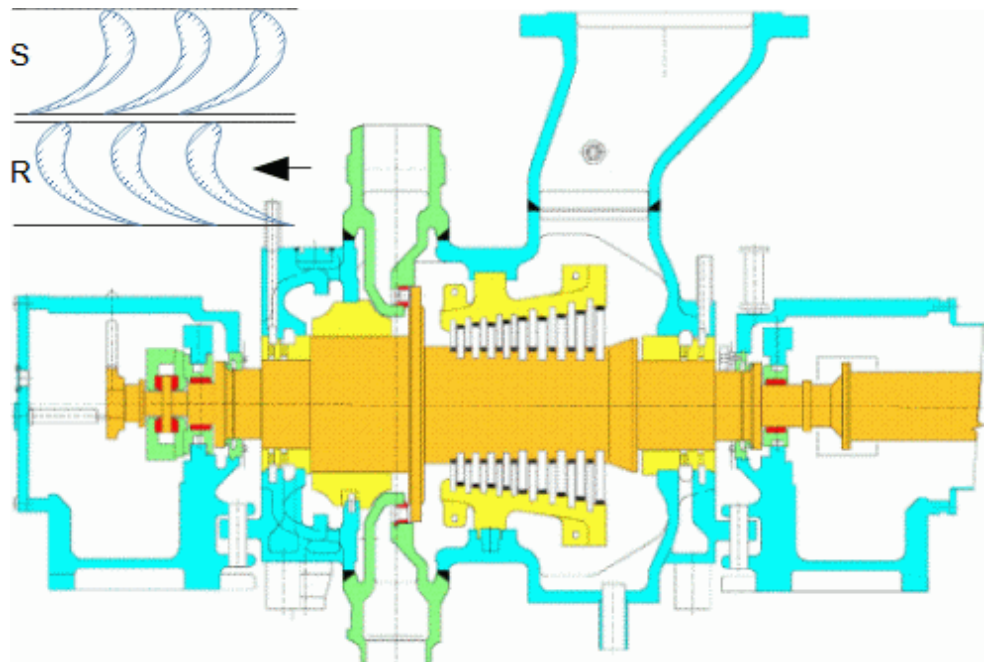


13: Laval turbine (single stage steam turbine)

a-nozzle (there may be several nozzles around the circumference for higher flow and power); b-rotor; c-outlet; d-gearbox; e-el. generator; f-direction of rotor rotation. 0-steam inlet; 1-space between rotor and nozzle; 2-steam outlet from rotor; 3-steam outlet, p [Pa] pressure.

Multi-stage steam turbine

Larger enthalpy differences are more advantageously processed by multiple stages in a multi-stage turbine. Each stage contains a stator row of blades attached to the casing (forming a series of nozzles spaced around the perimeter) and a rotor row of blades, see [Figure 14](#).

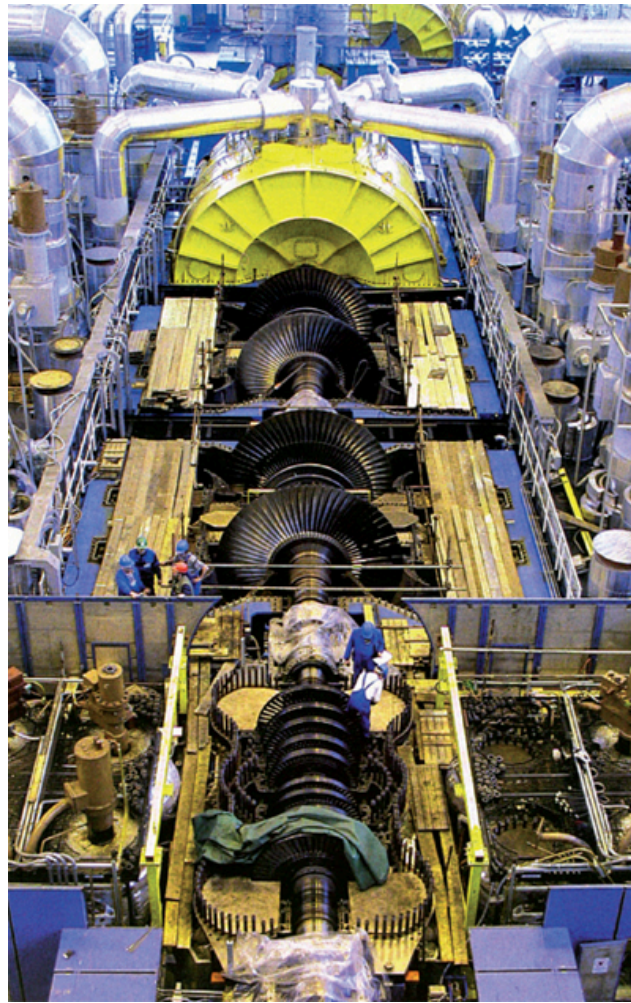


14: 6 MW multi-stage steam turbine

9980 min^{-1} , inlet parameters: 36,6 bar, 437 °C, outlet steam pressure 6,2 bar. S-stator blade row; R-rotor blade row. Manufactured by Alstom Brno branch (CZ) [Anon., 2009c].

Multi-casing turbine

Large power turbines are divided into several smaller turbines (casings) - this solves the problem of large bearing distances in the case of multi-stages or the problem of large volume flow. The turbine casings are arranged in series connected by couplings, or side by side without couplings, and the steam distribution between the bodies can be in series or parallel, such turbines are called multi-casing turbines, see [Figure 15](#).



15: Four-casing steam turbine at Temelín Nuclear Power Plant

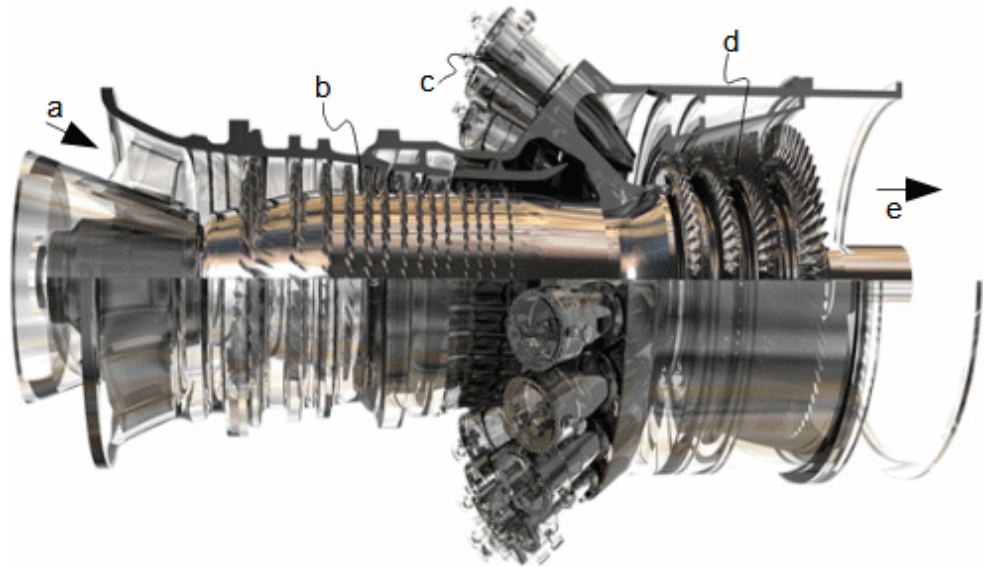
The length of turboset is 63 m, it means length including generator, the length of the rotor is 59,035 m (turbine rotor 36,45 with weight of 240 t) and its weight 326,4 t (2000 t is total weight of turboset), of which 93 t weighs the rotor of one low-pressure part [Anon. 2014]. 1x highpressure casing; 3x lowpressure casings. The last casing of turbine is closed. Made by Škoda (CZ) [Hlavatý and Krejčí 2007].

Gas turbines
Combustion turbine

The working fluid of gas turbines is gas or flue gas. The most common gas turbines are those with a combustion chamber (these are called combustion turbines). Combustion turbines contain a turbocompressor section and a turbine section. [Figure 16](#) shows a section of a combustion turbine. The turbocompressor compresses the intake air. In the combustion chamber, combustion of the fuel and compressed air takes place. The combustion produces hot exhaust gases (gas) which drive the turbine section of the combustion turbine. The power of the turbine section is used to drive the turbocompressor (most of the power) and an electrical generator or other equipment.

Jet engine
Thrust

Combustion turbines are also used to drive jet engines - in this case, the power of the turbine section is equal to the power input of the turbocompressor, and the rest of the enthalpy gradient contained in the exhaust gas is used for expansion in the engine nozzle and creates thrust on the reaction principle.



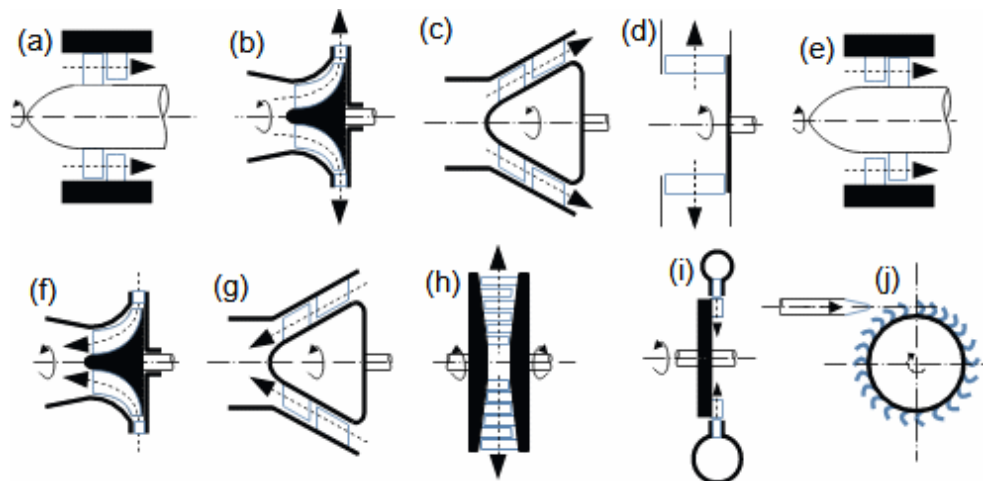
16: Combustion turbine GE-9F series

a-air intake; b-compressor stages; c-combustion chambers; d-turbine stages; e-exhaust gas outlet. Output power 300 MW [Anon. 2011]. Figure modified.

Nomenclature of turbomachines according to meridional flow direction

Axial, radial, diagonal
and tangential
turbomachines

Classification of turbomachines by a stream direction in relation to the axis of the shaft (Figure 17 – four main directions or meridional directions: axial, radial, mixed and tangential) informs about design of the machine. The predominant flow direction is usually reflected in the machine name.



17: Turbomachines according to the direction of meridional flow

(a) to (d) are pumps, compressors or fans; (e) to (j) are turbines. (a) axial; (b) radial – with axial inlet; (c) mixed flow; (d) radial – flow (centrifugal); (e) axial; (f) radial – with axial outlet; (g) mixed flow; (h) radial – flow (in case with alternate rotors rotating opposite); (i) radial – flow (centripetal); (j) tangential – (Pelton turbine).

Specific speed

Each direction of flow gives the turbomachine stage different characteristics and the most suitable type for a given application is therefore determined by its specific speed⁶ and operating parameters.

Construction features of turbomachines

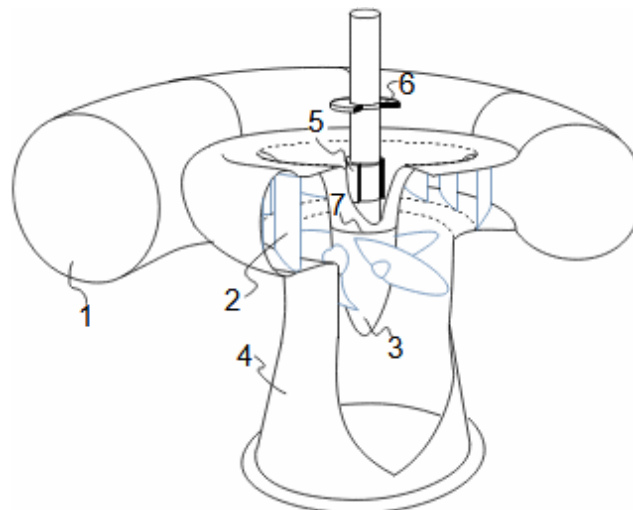
Branches

Kaplan turbine

Radial bearing

Axial bearing

Turbomachines contain flow and machine parts. In addition to the rotor, most turbomachines also contain inlet and outlet flow parts (branches), housing, bearings, shaft seals, etc. They often have fluid quality and quantity control. Figure 18 is a section of a Kaplan turbine that contains most of these parts.



18: Structural parts of Kaplan turbine

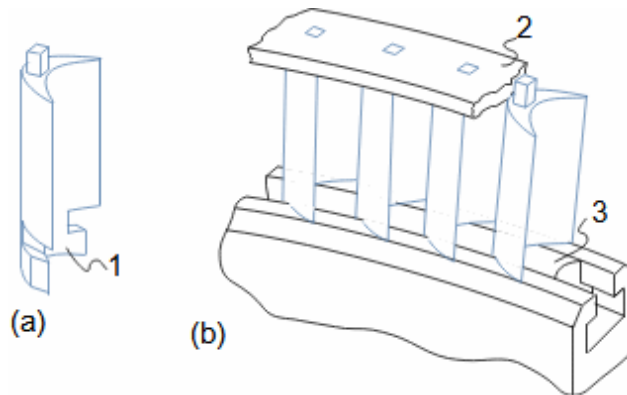
1-inlet of water into turbine through spiral casing (inlet branch); 2-stator blades (adjustable for flow control); 3-rotor (adjustable blades for efficiency control); 4-draft tube (outlet part or outlet branch); 5-radial bearing (captures forces perpendicular to axis of rotation); 6-axial bearing (captures forces parallel to axis of rotation); 7-rotor seal (shaft passage through the casing).

Blades

Blades³ are most often manufactured individually and are inserted into the rotor and stator through their root or attached to the rotor in some other way. The blade row is also referred to as the blade cascade. The representation of a cylindrical section on a certain radius of the blade array is referred to as a profile cascade and the shapes of the blade passages are clearly visible.

Blade cascade
Shroud

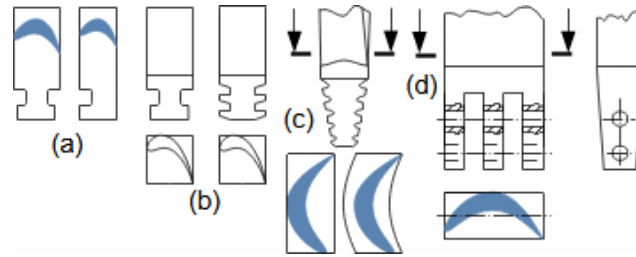
The blades in the blade cascade create a row of passages of the required dimensions and shapes, see [Figure 19](#). Some turbomachines have adjustable blades (this allows the size of the flow passage to be changed or completely closed), e.g. the Kaplan turbine. In some cases a shroud is placed on the blade tips.



19: Example of rotor disc structure of single-stage steam turbine (Laval turbine)
(a) blade; (b) formation of channels by using blades (blade passage); 1-blade root;
2-shroud (not necessarily); 3-spacer.

Blade roots
Force on blade
Blade oscillation
Spacer
Fatigue failure

The blade root ([Figure 20](#)) fixes the blade in the rotor or stator and captures the forces acting on the blade, which are mainly centrifugal force and the forces on the blades² from the fluid flow (the root type [Figure 20d](#) bears the greatest load). The root must also have a good damping function. Smaller natural frequencies of oscillation have roots that also integrate the spacer (a part that is inserted between adjacent blades to keep them at the required distance from each other), or even several blades are integrated on one large root (made of one piece or several blades and roots are welded together, etc.) [Škopek, 2007]. The root must be resistant to fatigue failure to prevent it from loosening from the grooves over time.

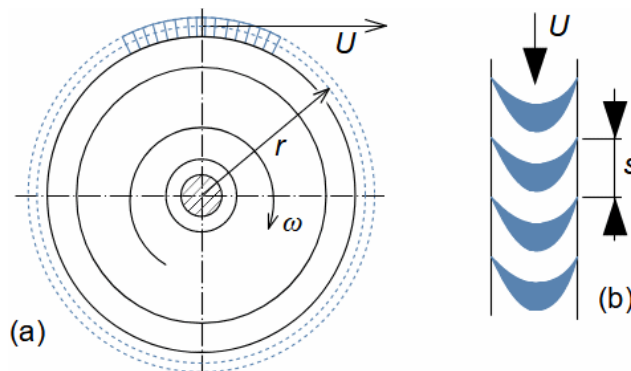


20: Basic types of blade roots

(a) examples of blade roots shapes common in drawn blanks; (b) single T-root and double T-root typical of milled profiles (mainly used in drum rotors); (c) tree root (this type and type (c) are used in disc rotors); (d) multi-finger pinned root. Blades with root types (a) and (b) are inserted tangentially into the rotor grooves as indicated in Figure 19, blades with root type (c) are inserted axially into the disc. Example manufacturing documentation of blade hinges can be found in [Škopek, 2007], [Michele, 1985], for example.

Profile cascade
Pitch of blades
Straight blade
Twisted blades

A section through the blade cascade is called the profile cascade (see Figure 21). As can be seen from the profile cascade, the size of the blade passages, or the distance between blades, or pitch, depends on the radius at which the cut is made. In this case, the blades are short relative to the diameter and the change in dimensions is not apparent; it is a so-called straight blade³. For higher efficiency, especially for longer blades, so-called twisted blades³ are used - their shape and size change along their length (e.g. Figures 1, 12, 11). Straight blades are often used in radial machines or as short blades in axial machines.

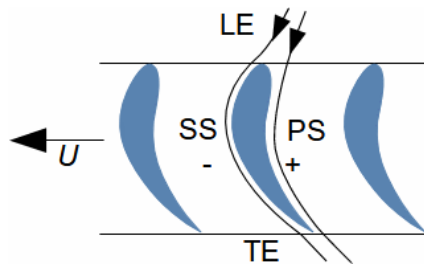


21: Laval turbine rotor disc

(a) turbine rotor mounted with blades; (b) developed cylindrical section through blade passages at radius r (profile cascade). r [m] mean radius of blades; s [m] pitch of blades; U [$\text{m}\cdot\text{s}^{-1}$] blade speed at radius r .

Profile
 Leading edge
 Trailing edge
 Suction surface
 Pressure surface

Figure 22 shows the names of the individual profile parts, which depend on the shape and orientation of the profile in the cascade. We refer to the inlet edges of the profiles as leading edge LE and the outlet edges as trailing edge TE. Along the curved surfaces of the profile, the pressure varies (see Aerodynamika profilů [Škorpík, 2022]) - we refer to the side of the profile with lower pressure as the suction surface of the profile SS and the side with higher pressure as the pressure surface of the profile PS.



22: Basic nomenclature of blade profile

Energy balance of turbomachine

Internal power
 Internal work
 Power input

One of the main parameters of the turbomachine is its internal power. Internal power is the power of the working fluid flowing through the turbomachine and is defined as the product of its internal work and mass flow, see Equation 23. If the working fluid consumes the work (working machine), the work will be negative and hence the value of P_i , but usually the negative sign is not given and the expression "power input" is used.

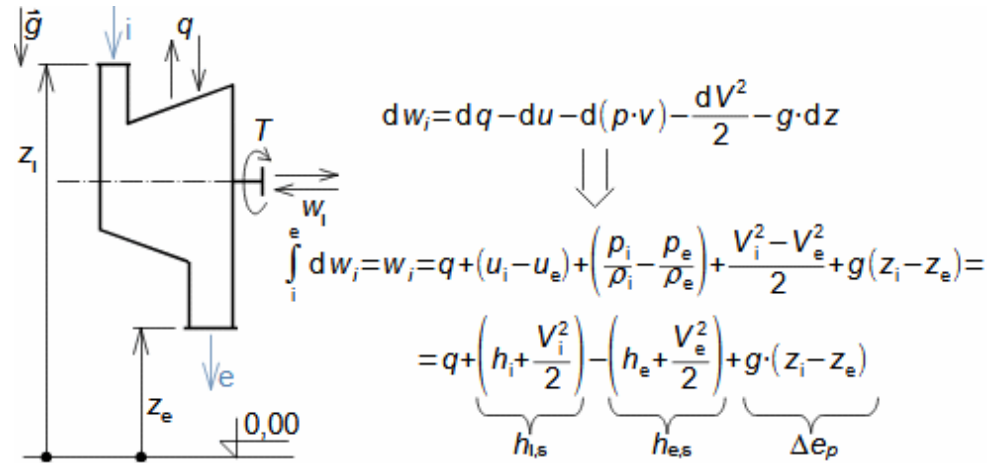
$$P_i = w_i \cdot \dot{m}$$

23: Internal power of turbomachine

P_i [W] internal power/power of turbomachine; w_i [$\text{J} \cdot \text{kg}^{-1}$] internal work of turbomachine; \dot{m} [$\text{kg} \cdot \text{s}^{-1}$] mass flow of working fluid through turbomachine.

Internal work
 First law of thermodynamics

To calculate the internal work of the turbomachine, the equation of the first law of thermodynamics for open system can be used, see Equation 24. This equation takes into account all the energy transformations in the working fluid that can occur in the turbomachine - it can do/consume work, it can be heated or cooled (heat can be shared with the working fluid through the walls of the machine or heat can be released in the working fluid e.g. by a chemical reaction), so the enthalpy, kinetic and potential energy of the working fluid can change.



24: First law of thermodynamics for open systems

ρ [$\text{kg}\cdot\text{m}^{-3}$] density; g [$\text{m}\cdot\text{s}^{-2}$] gravitational acceleration; u [$\text{J}\cdot\text{kg}^{-1}$] internal thermal energy; q [$\text{J}\cdot\text{kg}^{-1}$] heat of the working fluid shared with the surroundings (positive value: heat is delivered to the machine; negative value: heat is rejected from the machine); h [$\text{J}\cdot\text{kg}^{-1}$] enthalpy (static); h_s [$\text{J}\cdot\text{kg}^{-1}$] stagnation enthalpy of the fluid; e_p [$\text{J}\cdot\text{kg}^{-1}$] potential energy of the working fluid; T [$\text{N}\cdot\text{m}$] torque on the shaft. The index $_i$ indicates the input, the index $_e$ the output of the machine. This scheme of energy balance of an open system is taken from the article Technická termomechanika [Škorpík, 2024].

Hydraulic machines
Bernoulli equation
Internal losses
Head

A special form of the first law of thermodynamics for incompressible fluids is called the Bernoulli equation, see [Equation 25](#), and is used for calculating hydraulic machines. In this case, only pressure, kinetic, and potential energy transformations are acceptable, and transformations of other types of energy are considered as internal losses-hence the sum of the pressure, kinetic, and potential energy of the fluid is called the head of the fluid.

$$w_i = q + \frac{p_i}{\rho} + \frac{V_i^2}{2} - \frac{p_e}{\rho} - \frac{V_e^2}{2} + g(z_i - z_e) + (u_i - u_e) =$$

$$= \underbrace{\frac{p_i}{\rho} + \frac{V_i^2}{2} + g \cdot z_i}_{H_i} - \underbrace{\left(\frac{p_e}{\rho} + \frac{V_e^2}{2} + g \cdot z_e \right)}_{H_e} - L_w \quad -L_w = q + (u_i - u_e)$$

25: Bernoulli equation

H [$\text{J}\cdot\text{kg}^{-1}$] head; L_w [$\text{J}\cdot\text{kg}^{-1}$] internal losses on machine work.

Heat machine
Internal work

In heat machines, any kind of energy can be transformed, but the effect of potential energy changes is usually insignificant. Also, the effect of changes in the pressure energy and the internal thermal energy of the working fluid is not distinguished, and instead one operates with the quantity enthalpy, so that the First Law of Thermodynamics for open systems for these cases machines in the form of [Equation 26](#).

$$w_i = h_{i,s} - h_{e,s} + q$$

26: Specific internal work of heat turbomachine

The above special equations can be used in the design of a machine or its stage, for complete energy balances of technological units as well as for approximate calculations of basic machine parameters, as shown in [Problems 1](#) and [2](#).

Internal losses

Ideal internal work

The transformation of energy in the turbomachine induces internal losses⁵, denoted by the symbol L_w . The internal loss is the portion of the energy transformed to other than the required energy and arises, for example, from internal friction of the working fluid, mixing of cold and hot streams, heat exchange between streams, swirling, etc. The internal loss is reflected by a larger value of the internal thermal energy (enthalpy) and entropy of the working fluid at the machine outlet, compared to the case of ideal energy transformation in the turbomachine, see [Problem 2](#). This means that the internal losses are the difference between the ideal internal work of the machine and the actual internal work of the machine, see [Formula 27](#), where the internal ideal work of the machine is the work of the machine without internal losses (hence the index w at the letter L). The ideal work of hydraulic machines corresponds to the value of the change in the head of the fluid ($w_i = \Delta H$). The ideal work of thermal machines is usually isentropic w_{is} or polytropic reversible change w_{pol} .

$$L_w = w_{id} - w_i$$

27: Internal efficiency of turbomachine

w_{id} [$J \cdot kg^{-1}$] ideal internal work of turbomachine.

Internal efficiency

The internal efficiency η_i defines the efficiency of energy transformation inside the machine by comparing the actual internal work of the machine w_i with the internal ideal work of the machine w_{id} , see [Formula 28\(a, b\)](#).

$$(a) \eta_i = \frac{w_i}{w_{id}} \quad (b) \eta_i = \frac{w_{id}}{w_i}$$

28: Internal efficiency of turbomachine

(a) internal efficiency of turbines; (b) internal efficiency of working machines. η_i [1] internal efficiency.

Hydraulic efficiency

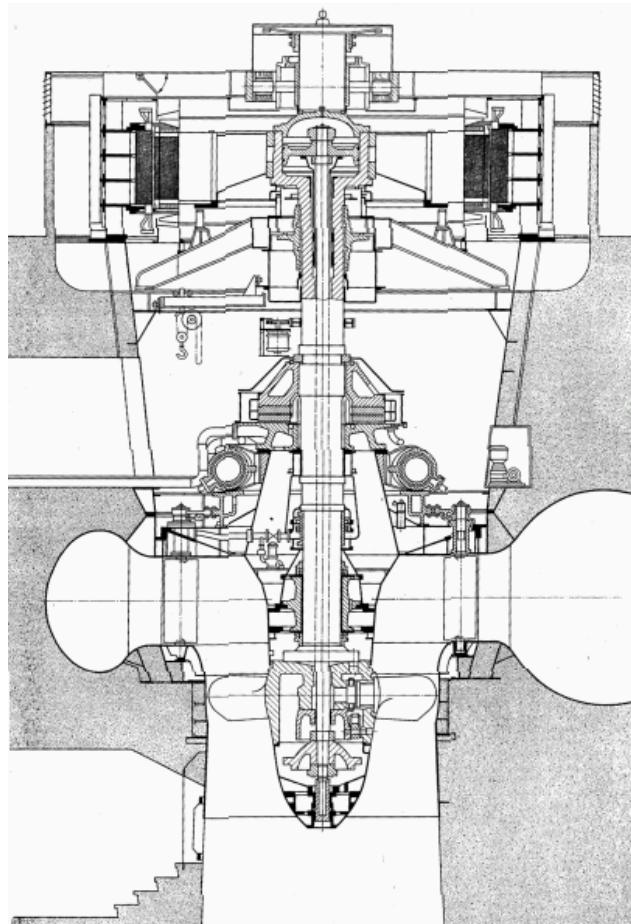
Thermodynamics efficiency

It is customary to refer to the internal efficiency of hydraulic machines as hydraulic efficiency and that of heat machines as internal thermodynamic efficiency. The word "internal" is omitted for wind turbines and propellers.

Energy balance of turboset

Turboset
Kaplan turbine

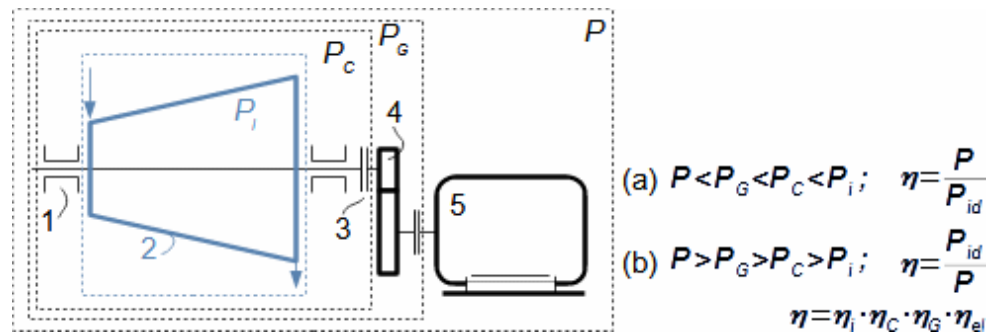
Turbomachines are always connected to another machine (e.g. turbine/generator, pump/motor, etc.). Machine assemblies with a turbomachine are called turbosets, see [Figure 29](#). [Figure 30](#) shows a typical schematic of the turboset containing, in addition to the turbomachine, a gearbox and an electric generator. Also indicated in this figure is the indicated power in the individual parts of the machine, which is progressively reduced from the internal power of the machine (the power of the working fluid in the flow section) by losses in the individual parts of the turboset, see [Problem 3](#).



29: Turboset of Kaplan turbine and turbogenerator
Manufactured by Voith [Miller et al. s. 591].

Nominal and optimal
power

The parameters of the turboset are indicated on its label. This label shows the nominal power P_n (reference power, usually maximum) and the optimum power P_{opt} at which the machine achieves maximum efficiency.



30: Turboset efficiency and power input/output

(a) power of turboset; (b) power input of turboset. 1-machine bearing; 2-machine interior; 3-coupling; 4-gearbox; 5-generator/drive. P_C [W] input/output power at coupling; P_G [W] input/output power of gearbox; P [W] input/output power at generator/drive contacts; P_{id} ideal turboset power - all energy transformations in turboset are lossless; η [1] turboset efficiency; η_c [1] turbomachine mechanical efficiency; η_g [1] gearbox efficiency; η_{el} [1] generator/drive efficiency.

Agreement

The indication of the efficiency of the turboset and its performance is an important agreement parameter. Equally important are the efficiencies of the individual components of the turboset for the agreement between the subcontractors and the final contractor of the turboset, in order to trace which of the subcontractors failed to meet the technical criteria if the turboset as a whole did not meet the parameters in the agreement.

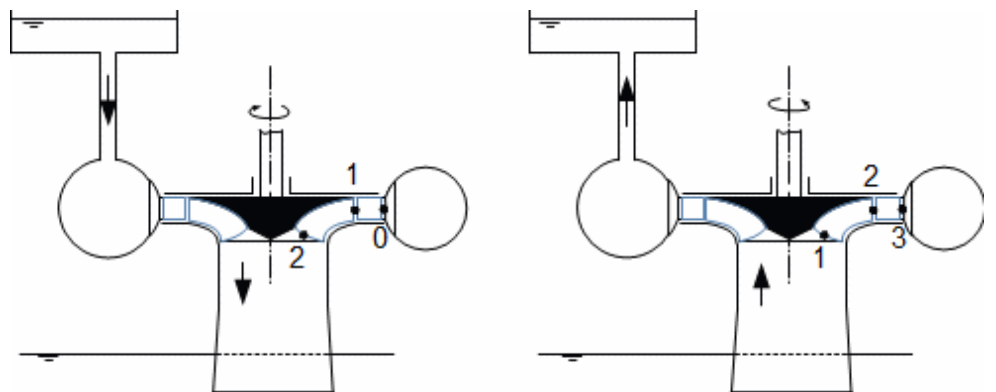
Turbomachine stage

Stator

Rotor

Francis turbine

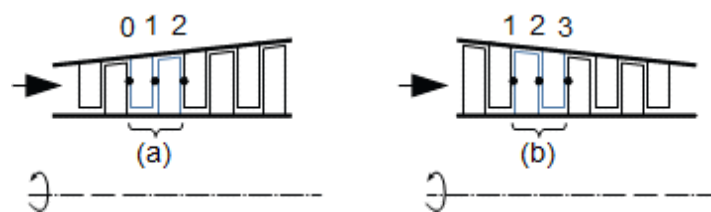
The turbomachine stage contains the stator (stator row of blades) and the rotor (rotor row of blades). The Francis pump turbine stage (reverse turbine) is shown in Figure 31 as an example of the turbomachine blade stage composition. The turbine stage is made up of first the stator row of blades then the rotor row, the reverse is true for the working machines.



31: Turbomachine stage

Energy transformation
Marking of states

The energy of fluid can only be transformed into work in the rotor, therefore index 1 before the rotor and index 2 after the rotor is used for the working fluid state. In turbines, the fluid state before the stator is denoted by index 0. For working machines, the fluid state after the stator is indicated by index 3. For multi-stage machines, the method of marking within the stage is identical, see [Figure 32](#).

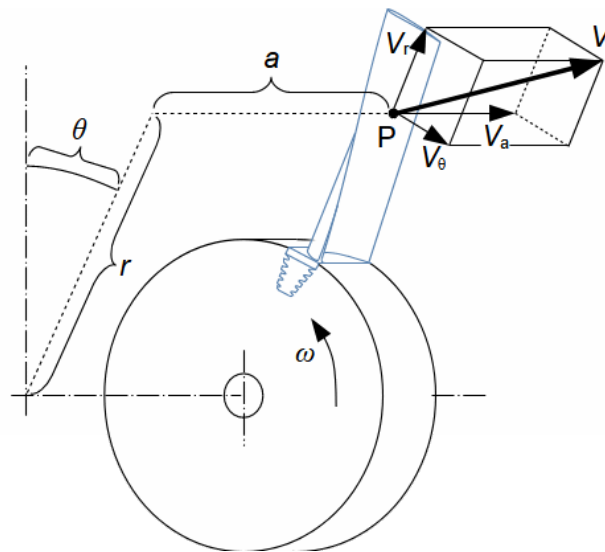


32: Example of working fluid state marking on multi-stage turbomachine
(a) turbine stage; (b) turbocompressor stage.

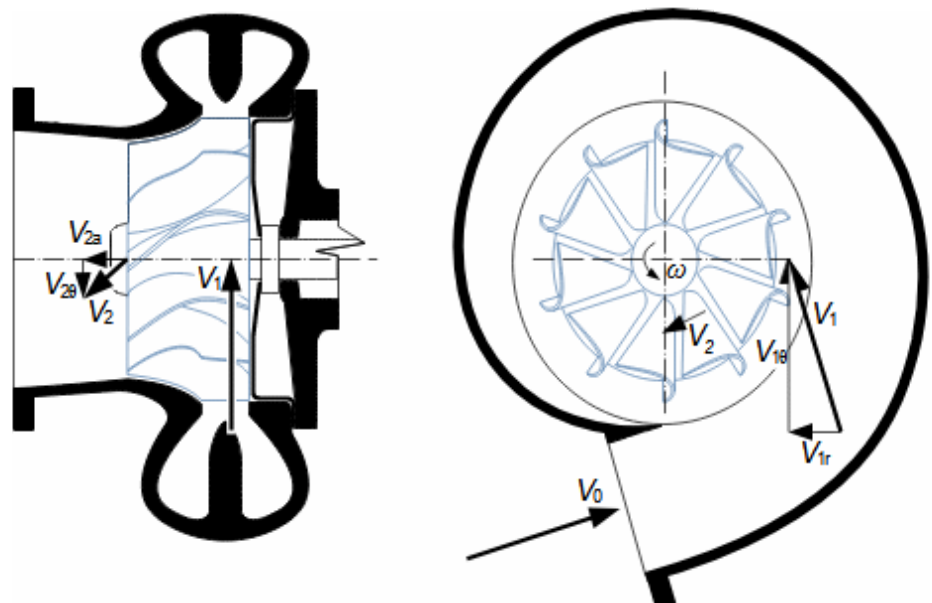
Velocity triangle

Absolute velocity
Cylindrical coordinate
system

In [Equations 25](#) and [26](#) of the energy balance applied to the stage, the velocities V in front of and after the blade row stand out. The fluid velocity V is called the absolute velocity and can project in three directions because it is a flow in space. In the case of turbomachinery, a cylindrical coordinate system is used to denote these components, which is clearer than a rectangular coordinate system for describing motion about an axis, as shown in [Figure 33](#). The component of velocity perpendicular to the axial direction is called radial- r , the component of velocity in the direction of rotation is called tangential- θ , and the component of velocity in the direction of the axis is called axial- a . The absolute velocity is therefore the vector $V^{\rightarrow}(V_r, V_{\theta}, V_a)$, in the following the arrow denoting the vector is not shown for clarity. [Figure 34](#) shows an example of the absolute velocities of the working gas in front of and behind the rotor of a turbocharger turbine and their components according to the proposed orientation of the cylindrical coordinate system.



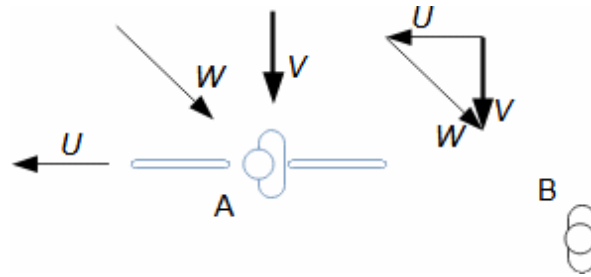
33: Absolute velocity in a cylindrical coordinate system
P-point at which investigated velocity V ; θ [°] azimuth.



34: Example of absolute velocities around rotor of turbocharger turbine

Relative velocity

The absolute fluid velocity V is the vector sum of the relative fluid velocity W and the blade speed U . The relative fluid velocity W is the fluid velocity observed by an observer moving with the rotor of the stage. The relative velocity can have three spatial components. In order to clarify the concept of relative velocity, there is the [Figure 35](#) showing a moving cyclist A at speed U and a stationary observer B. While the stationary observer observes the absolute direction and magnitude of the wind V , the cyclist observes the direction and magnitude of the wind W , which we refer to as relative, i.e., relative to the moving point with respect to the reference (stationary) point.



35: Relative velocity

U [$\text{m}\cdot\text{s}^{-1}$] cyclist speed; W [$\text{m}\cdot\text{s}^{-1}$] relative wind speed.

Blade speed

The blade speed is defined as the product of the radius of rotation r and the angular velocity ω (see Equation 36) and has no components in the axial and radial directions. The blade speed lies in a plane perpendicular to the axial direction.

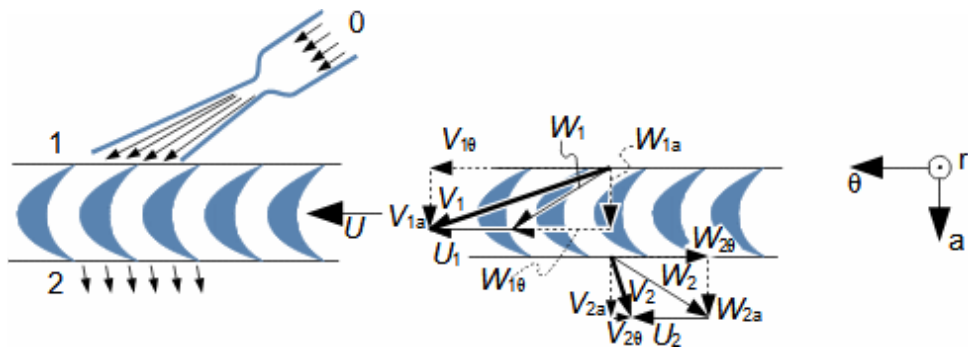
$$U = 2\pi \cdot r \cdot N = \omega \cdot r$$

36: Blade speed

N [s^{-1}] Rotor speed.

Laval turbine

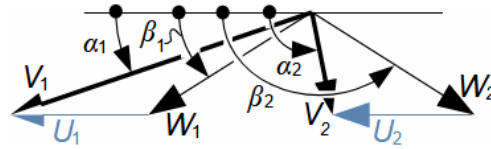
A graphical representation of the absolute and relative fluid velocity and blade speed is called a velocity triangle. Figure 37 shows such velocity triangles for the rotor of the Laval turbine of Figure 13, where the working fluid (steam) inlet to the rotor blade passages is at velocity V_1 and outlet and velocity V_2 .



37: Velocity triangle of Laval turbine rotor

Velocity angles

The velocity triangle is not usually drawn together with the profile cascade, but is shown separately for clarity and calculation purposes. The angles of the individual velocities are also plotted, as shown in Figure 38, which also presents additional rules for its construction. For example, the inlet and outlet velocity triangles are drawn in the plane of the flow. The positive direction of the individual velocity components is in the direction of the blade speed. The angles are quoted counterclockwise for ease of calculation using goniometric functions, but other quoting of angles is possible see [Kadrnožka 2003, p. 26].



38: Velocity triangle

α [°] angle of absolute velocity; β [°] angle of relative velocity.

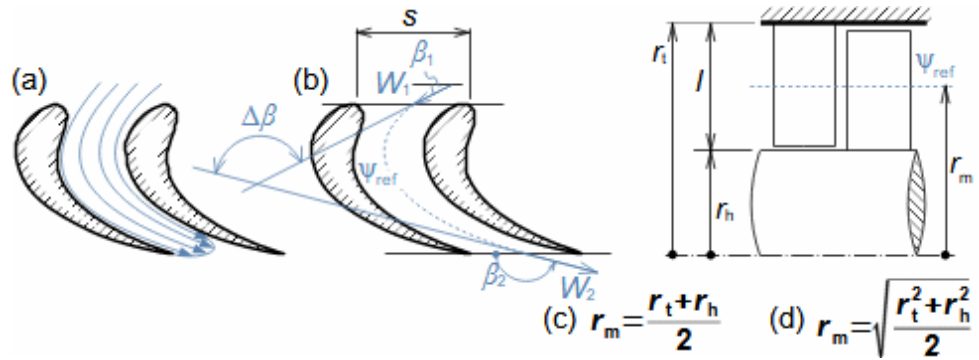
Design of turbomachine stage

Velocity triangle
Blade passage

When calculating the stage of the turbomachine, the values and directions of the velocities in the velocity triangle are important for the design of the shape of the blade passages, respectively the blades - if the direction is known, the curvature of the passages can be designed, if the change in velocity is known, it can be designed whether the passage should be narrowed or widened, etc. The velocity triangle is valid for a specific point under investigation in the working fluid volume in the machine. A neighbouring point will already have a slightly different velocity triangle, so when designing the turbomachine stage, we approach a certain level of simplified flow description according to the design accuracy requirement. Depending on the level of simplification we talk about 1D, 2D and 3D calculation.

1D calculation
Mean velocity
Mean blade radius
Mean square radius of blade
Straith blade

In a 1D calculation, the real spatial velocity field in the blade passage is replaced by a single reference streamline with a mean flow velocity, see [Figure 39\(a\)](#). The reference streamline passes through the centre of the blade passage and is located at the mean or quadratic radius of the blade as decided by the designer, see [Figure 39\(b\)](#). Many simplifications are introduced in the calculation so that the calculation is simple but sufficiently representative over the entire volume of the stage. It is used when designing the blade passage shape of a machine with straight blades, i.e. where the blades are short and there is no significant difference in blade speed between the root and tip, see [Problem 4](#).



39: Diagram of 1D flow through stage at medium radius

(a) actual flow in the blade series; (b) simplification to 1D flow; (c) equation for mean radius of blades; (d) equation for mean square radius of blades (mean square radius is the radius at which the area between r_t and r_m is as large as the area between r_m and r_h). l [m] blade length; $\Delta\beta$ [°] required curvature of the flow. ψ -streamline. The index $_{ref}$ denotes reference, $_t$ radius of the blades at the tip (tip), $_h$ radius at the base of the blades (hub), index $_m$ denotes mean. The derivation of the mean square radius equation is shown in [Appendix 6](#).

Mean velocity

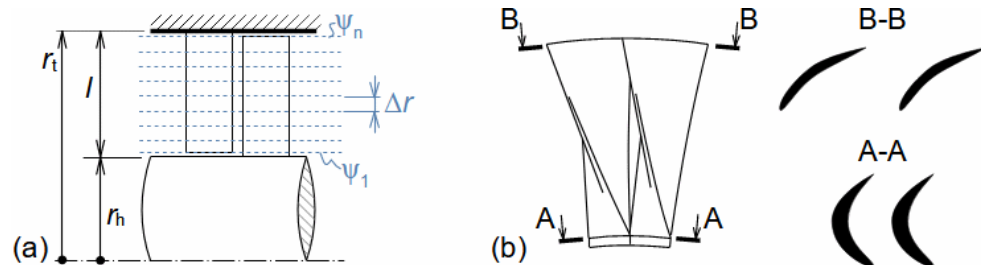
The mean velocity in the blade passage can be determined from the continuity equation, or from the mean value of the kinetic energy of the working fluid in the passage, see the definition of mean velocities in the article *Vnitřní tření tekutiny a vývoj mezní vrstvy* [Škorpík, 2023].

2D calculation

Twisted blade

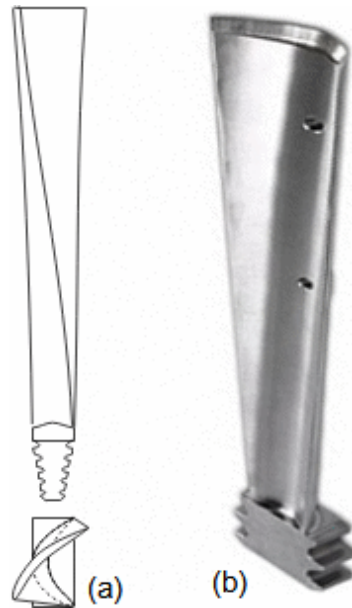
Elementary stage of turbomachine

The 2D calculation follows a similar procedure as in the previous case (replacing the real flow field with a mean velocity flow streamline), except that the calculation of the velocity triangle is performed on several diameters, see [Figure 40\(a\)](#). This calculation method is mainly used in the calculation of turbomachinery stages, with the emphasis on achieving the best possible shape of the blade passage respecting the spatial character of the flow (increasing the pitch with the investigated radius and increasing the blade speed). The calculation is the basis for the shape of the blade passage at each radius of the twisted blades, see [Figure 41\(b\)](#), or of the blades of radial stages with axial part, see [Problem 5](#).



40: Elementary stage of turbomachine

(a) division of blade into n computational elements - calculation on specific radius is therefore called elementary stage of turbomachine; (b) example of changes in shape of blade passage between root and tip of twisted blades - both pitch and shape of blade passage changes according to results of velocity triangles for given elementary stage. n -number of elements; Δr [m] element height of stage.



41: Examples of twisted steam turbine blades

(a) change in shape of twisted blade designed to take into account spatial character of flow in the stage; (b) example of twisted blade of steam turbine (photo Wiromet s.a.).

3D calculation
FEM

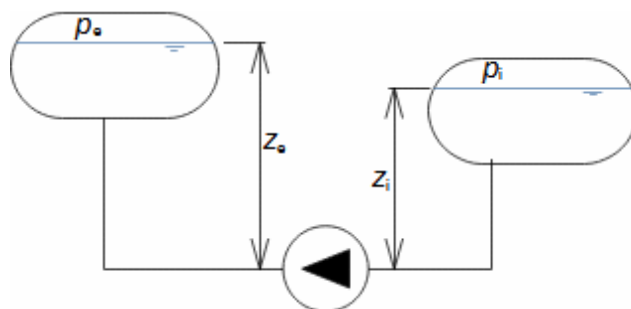
The 3D calculation is a complex numerical calculation of the turbomachine stage using advanced finite element methods (FEM) software. It usually takes into account velocity changes in the vicinity of the profiles (boundary layer effects). Before applying the 3D calculation, the approximate geometry of the stage calculated from the 1D or 2D calculation is already known.

Problems

Problem 1:

Internal work
Internal power input

$20 \text{ t}\cdot\text{h}^{-1}$ of water is pumped from the lower tank to the upper tank by a turbopump. The pressure in the lower tank is 1 bar, the pressure in the upper tank is 40 bar, the difference in level is 7 m. What is the approximate internal work and the approximate internal power input of the pump? The solution to the problem is shown in [Appendix 1](#).



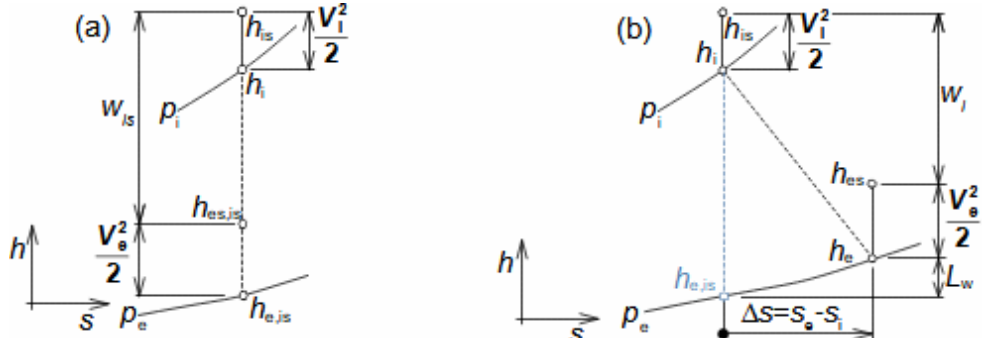
§1	entry:	$m; p_i; p_e; \Delta z$	§3	proposal:	$V_i; V_e$
§2	read off:	$g; \rho$	§4	calculation:	$w_i; P_i$

The procedure for solving Problem 1. Symbol descriptions are in [Appendix 1](#).

Internal work
Internal losses
Internal efficiency

Problem 2:

Steam enters the steam turbine at a pressure of 36.6 bar and a temperature of 437 °C. The pressure at the turbine outlet is 6,2 bar and the internal work of the turbine is 410 kJ·kg⁻¹. Find the internal losses and the internal efficiency of the turbine - the turbine is thermally well isolated ($q \approx 0$), so the comparative ideal process is isentropic expansion. The solution to the problem is shown in [Appendix 2](#).



(a) changes in steam state during isentropic expansion; (b) changes in steam state during actual expansion. s [J·kg⁻¹·K⁻¹] entropy. The index _{is} denotes isentropic change.

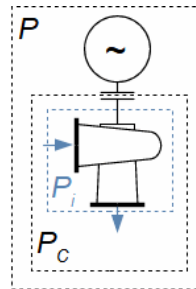
§1	entry:	$p_i; t_i; p_e; w_i$	§3	calculation:	$w_{id}; L_w; \eta_i$
§2	read off:	$s_i; h_i; h_{e,iso}$			

The procedure for solving Problem 2. Symbol descriptions are in [Appendix 2](#).

Turboset

Problem 3:

Calculate the power output of a water turbine generator. The internal power of the turbine is 15 MW, the efficiency of the turbine at the coupling is 97,5 %, the efficiency of the generator is 97 %. The solution of the problem is shown in [Appendix 3](#).



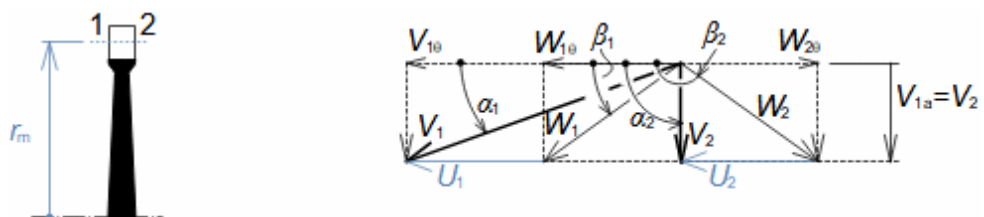
§1	entry:	$P_i; \eta_C; \eta_{el}$	§2	calculation:	$P_C; P$
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The procedure for solving Problem 3. Symbol descriptions are in [Appendix 3](#).

Laval turbine
Velocity triangle

Problem 4:

Find the velocity triangle of a Laval turbine at the mean radius of the blades, which is 80 mm. The rotor speed is 29 625 min⁻¹. The other parameters of the velocity triangle at the central radius are $U_1=U_2$, $V_1=530$ m·s⁻¹, $V_{20}=0$ m·s⁻¹, $W_1=W_2$. The solution of the problem is shown in [Appendix 4](#).



§1 entry:	$r_m; N; V_1; V_{20}$	§4 calculation:	$V_a; W_a; V_2$ (Pythagorean theorem)
§2 proposal:	velocity triangle shape	§5 calculation:	$W_{1,2}$ (Pythagorean theorem)
§3 calculation:	$U_{1,2}; W_{10}; W_{20}; V_{10}$ (Eq. 36)	§6 calculation:	$\alpha_1; \beta_1; \beta_2$ (goniometric functions)

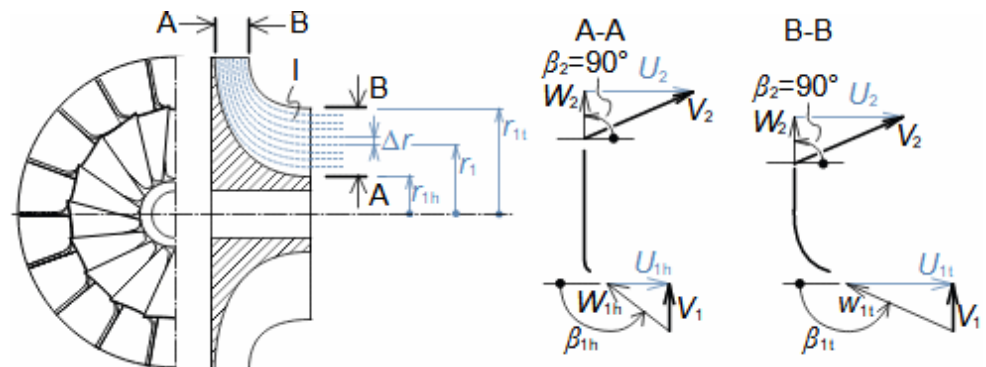
The procedure for solving Problem 4. Symbol descriptions are in [Appendix 4](#).

Problem 5:

Radial compressor
Inducer

Design the angles of the blades respectively the relative velocities of the inducer compressor impeller at the selected radii (the outlet angle of the blades respectively the relative velocities at the outlet is the same at all radii investigated - $\beta_2=90^\circ$). The angle of the inducer along the leading edge height is approximately equal to the angle of the inlet relative velocity, see attached figure. The radius of the blades at the rotor inlet is 196,2 mm at the tips and 61,2 mm at the roots. The mass flow through the rotor at $10\,000\text{ min}^{-1}$ is $27,2\text{ kg}\cdot\text{s}^{-1}$. The inlet gas density is $1,2\text{ kg}\cdot\text{m}^{-3}$.

The solution to the problem is shown in [Appendix 5](#).



I-inducer; A-A section through blade at its base; B-B section through blade at its tip; Δr [mm] height of elementary stage.

§1 entry:	$r_{1t}; r_{1h}; N; m; \rho_1$	§4 calculation:	$r_{1h}; U_{1h}$ (Eq. 36)
§2 entry:	$A_1; V_1$ (eq. continuity)	§5 calculation:	β_{1h} (goniometric functions)
§3 proposal:	Δr		

The procedure for solving Problem 5. Symbol descriptions are in [Appendix 5](#).

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