ESSENTIAL EQUATIONS OF TURBOMACHINES

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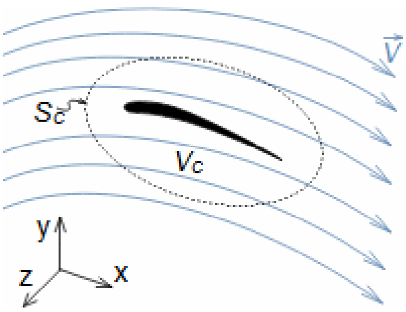
Equation for calculating forces acting on machine surfaces from fluid flow

The forces acting from the fluid flow on the machine surfaces can be determined from the momentum change theorem. Its special form is also used to calculate the forces acting on the blades in the blade row from the fluid flow, a classical problem in turbomachinery.

Theorem of momentum change
Control volume
Pressure forces
Weight
Force from bodies

The forces acting from the fluid flow on the machine surfaces can be determined from the momentum change theorem (Newton's second law). According to the momentum change theorem, the change in fluid momentum over time is equal to the sum of the external forces acting on the fluid in the control volume. In the case of applying this law to a fluid enclosed in a control volume V_C (Figure 1), the external forces considered are: the pressure forces from the surrounding fluid at the boundaries of the control volume F_p, the weight of the fluid in the control volume F_h, and the forces exerted by the bodies inside and at the boundaries of the control volume F_b. The change in momentum of the fluid inside the control volume is also equal to the difference of the product of velocity and mass flow between the inlet and outlet of the control volume [Bathie, 1984], [Kadrnožka 2003].



$$\frac{d\bar{M}}{dt} = \bar{F}_b + \bar{F}_h + \bar{F}_p = \int_{S_c} \bar{V}\, d\dot{m}$$

$$\bar{F}_h = \int_{V_c} (\bar{g} + \bar{a}_r + \bar{a}_c)\, dm$$

$$\bar{F}_p = \int_{S_c} p\, d\bar{S}_c$$

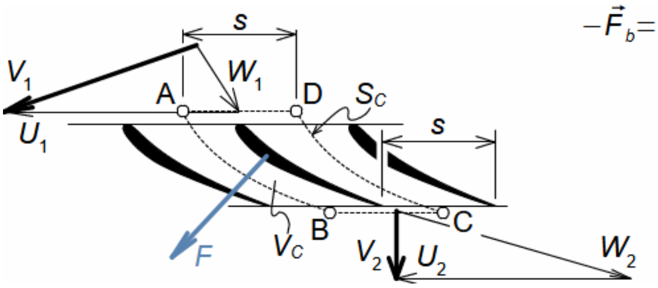
1: Theorem of momentum change

V_C [m^3] control volume; S_C [m^2] area of control volume boundary; V [m·s^{-1}] velocity of working fluid; M [N·s] momentum of fluid inside the control volume; t [s] time; F_b [N] resultant of forces acting on working fluid from bodies inside and on control volume boundary; F_h [N] weight of working fluid inside control volume; F_p [N] forces from pressure of surrounding fluid on surface of control volume; $\dot{m}$ [kg·s^{-1}] mass flow; g [m·s^{-2}] gravitational acceleration; a_r [m·s^{-2}] centrifugal acceleration; a_C [m·s^{-2}] Coriolis acceleration; p [Pa] pressure; m [kg] mass. The derivation of the equation assuming steady fluid flow through the control volume is shown in Appendix 11.

Force on blade
Control volume
Relative velocity
Pitch of blades
Velocity triangle

When calculating turbomachine blade forces, define control volume boundaries where parameters for momentum change theorem are known. Therefore, the control volume in Figure 2 is defined to pass through the center of the blade channel, or the boundaries AD and BC are spaced apart by the pitch of blades[1.]. The boundaries AD and BC are the expected streamlines of the relative velocities W of the velocity triangle[1.] of this blade row. The blade passages are equal in a single blade row, so that the action of the pressure forces at the AB boundaries cancel with the action of the pressure forces at the DC boundaries. The integration of the product of the absolute velocity V and the mass flow is also cancelled at these boundaries, see Equation 2.

$$-\vec{F}_b=\vec{F}=\vec{V}_1\cdot\dot{m}-\vec{V}_2\cdot\dot{m}+\vec{F}_h+\vec{F}_p$$



2: Force on blade from fluid flow

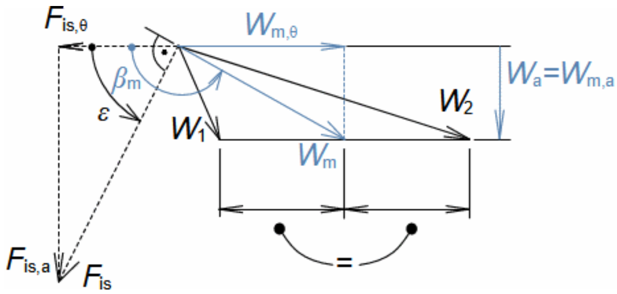
Both velocities and forces are vector quantities, but the arrow above the velocity symbols in the velocity triangle is usually not shown. F [N] resultant of forces acting on blade; W [m·s^{-1}] relative velocity; U [m·s^{-1}] blade speed; $m^{\bullet}$ [kg·s^{-1}] amount of working fluid flowing through control volume; s [m] pitch of blade. The derivation of the equation assuming steady fluid flow through the control volume is shown in Appendix 12.

Cylindrical coordinate system
Radial force
Tangential force
Axial force
Thrust bearing

Typical for the investigation of forces in the turbomachine stage is the use of cylindrical coordinate system[1.]. The force F in the cylindrical coordinate system has three spatial components, namely a component in the radial direction F_r, in the tangential direction F_θ (this force produces torque) and in the axial direction F_a (it causes stress on the rotor in the axial direction and is collected by the thrust bearing) - these force components are abbreviated as radial, tangential and axial forces.

Mean aerodynamic velocity

The force acting on the blade is approximately perpendicular to the mean aerodynamic velocity in the blade row W_m, which is the mean of the relative velocities at the inlet W_1 and outlet W_2. Respectively, it can be shown (see Equation 3) that the resulting force on the blade from the incompressible fluid flow F is perpendicular to the mean aerodynamic velocity w_m in lossless flow.



$$\vec{W}_m=\frac{1}{2}(\vec{W}_1+\vec{W}_2)$$

$$\tan\varepsilon=\frac{1}{-\tan\beta_m}\Rightarrow F_{is}\perp W_m$$

3: Definition of the mean aerodynamic velocity in the blade row and its relation to the force vector acting on the elementary blade (blade length dr)

W_m [m·s^{-1}] mean aerodynamic velocity in the blade series; β_m [°] angle of mean aerodynamic velocity; ε [°] angle of resultant force. This equation is derived for the elementary blade length Δr and the axial blade grid in Appendix 13 and its validity is limited to incompressible flow without losses (isoentropic - index $_{is}$).

Equations for calculating energy distribution in turbomachine stage

Meridian direction

The design of the energy distribution or transformation in the turbomachine stage is based on two directions. In the direction perpendicular to the meridian direction[1.], the Euler work distribution, which is the local value of the internal work, is designed. In the meridional direction, the design of the reaction stage, which describes the distribution of energy transformations between the stator and the rotor of the stage, decides the characteristics of the stage.

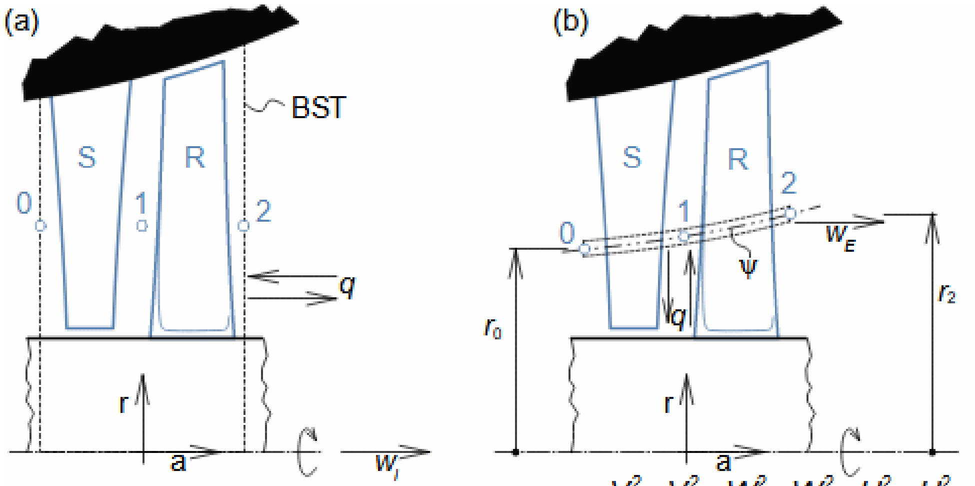
Euler work

Internal work of stage

Losses

Euler turbomachinery equation

The Euler work is the fluid work transferred to the blades in the surroundings of the streamline under investigation, see Figure 4(b). The difference with the internal work[1.] of the stage w_i is that the internal work of the stage is the average work of all the working fluid flowing through the stage (including gaps) and can be determined from the complete energy balance of the stage, see Figure 4(a). So some of the fluid will do more Euler work than others, but their average is w_i. For real stages, the largest Euler work is in the core of the flow (at the mean diameter of the blades), where the losses are smallest. Conversely, at the edges of the blades, or near their hubs and tips, the Euler work is smallest due to high frictional losses and internal leakage. The Euler work can be determined from the velocity triangles on the streamline under investigation, see Equation 4(c) - Euler turbomachinery equation.



$$\text{(c)}\quad w_E=U_1\cdot V_{1\theta}-U_2\cdot V_{2\theta}=\omega(r_1\cdot V_{1\theta}-r_2\cdot V_{2\theta})=\frac{V_1^2-V_2^2}{2}+\frac{W_2^2-W_1^2}{2}+\frac{U_1^2-U_2^2}{2}$$

4: Difference between Euler work and internal work of stage

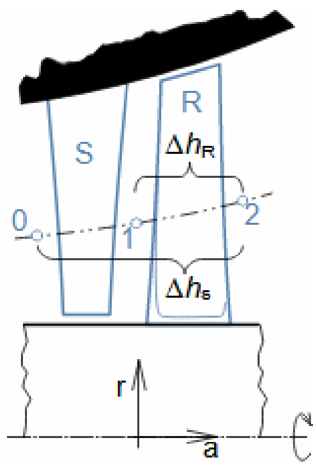
w_i [J·kg^{-1}] internal work of stage; w_E [J·kg^{-1}] Euler work in surroundings of investigated streamline; q [J·kg^{-1}] shared heat with surroundings; ω [°] angular speed. BST-stage boundary; S-stator blade row; R-rotor blade row, ψ-streamline. The derivation of the Euler turbomachinery equation for the assumption of stationary flow and no weight effect is shown in Appendix 14 or [Ingram, 2009].

Euler efficiency
Internal efficiency

Similarly, the stage efficiency can be determined in two ways, either to the Euler work (Euler efficiency) or to the internal work (internal stage efficiency), as done in Problem 4.

Reaction
Stator
Rotor
Hydraulic machine

Reaction is the ratio between the change in the static enthalpy in the rotor blade row and the change in the stagnation enthalpy of the stage (Formula 5), or the change in the static enthalpy of the stage - depending on convention [Kadrnožka, 2004], [Japikse, 1997], [Bathie 1984], [Ingram, 2009]. Thus, it describes the distribution of the energy transformation between the stator and rotor blade rows of the stage.



$$\text{(a)}\quad R=\frac{\Delta h_R}{\Delta h_s}$$

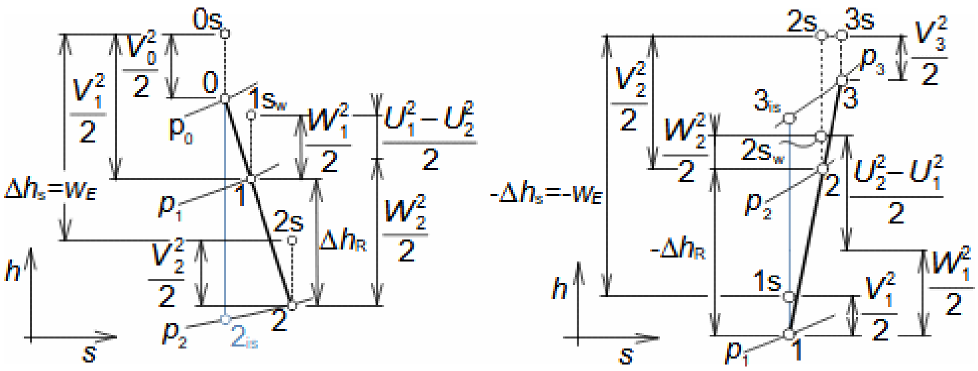
$$\text{(b)}\quad R\approx\frac{\Delta p_R}{\Delta p_s}$$

5: Definition of reaction

(a) definition of reaction; (b) simplified reaction formula for hydraulic machines, where approximate equality of enthalpy and pressure energy changes can be assumed ($\Delta h \approx \Delta p \cdot \rho^{-1}$). Δh_s [J·kg^{-1}] difference between stagnation enthalpy at inlet to stage and outlet from stage; Δh_R [J·kg^{-1}] difference between static enthalpy at inlet to rotor blade row and outlet from rotor blade row; Δp_s [Pa] difference between stagnation pressure at inlet to stage and outlet from stage; Δp_R [Pa] difference between pressure at the inlet to rotor blade row and outlet from rotor blade row; ρ [kg·m^{-3}] density.

h-s diagram of stage

Reaction is determined to a specific streamline (radius) similar to Euler work. To calculate the reaction, it is important to know the construction of the *h-s* diagram, from which the differences in specific enthalpies Δh_s and Δh_R can be determined (see Problem 5, Problem 6). *h-s* diagrams and a description of their construction are given in Figure 6. In the case of hydraulic machines, the required pressure differences Δp_s and Δp_R can also be determined from Bernoulli's equation for relative flow, see Problem 7 and Problem 6.

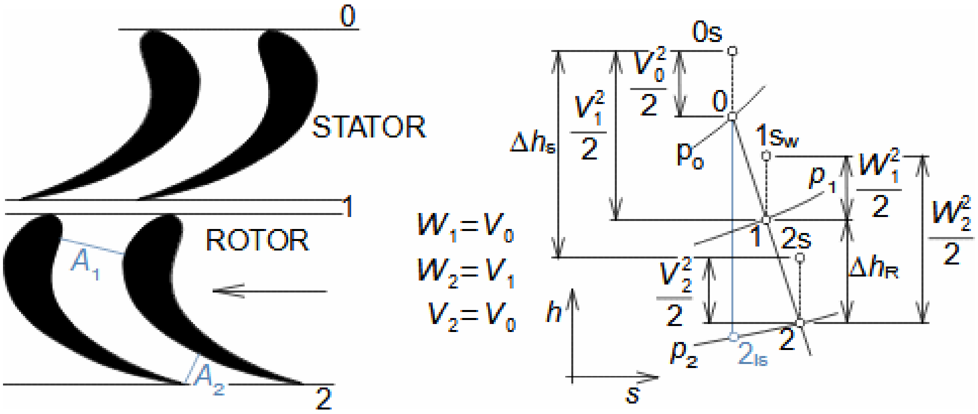


6: *h-s* diagrams of turbomachine stages

left-turbine stages; right-working machine stages. These *h-s* diagrams are constructed under the assumption of adiabatic flow without gravity effect. 1s$_w$, 2s$_w$ denote the overall condition with respect to the relative motion at the rotor inlet and outlet. A detailed description of the construction of the *h-s* diagrams is shown in Appendix 15.

Mean radius
Absolute velocity
Relative velocity
Losses

Most stages are designed with a variable reaction over the height of the blades, while a common requirement in stage design is a reaction at a mean radius of about 0.5 (even a little higher for radial stages due to the difference in blade speeds on the rotor), since at maximum Euler work the absolute velocities in the stator passages are about the same as the relative velocities in the rotor passages, and hence the distribution of losses between stator and rotor is uniform, see Figure 7.



7: Example of turbine axial stage blade passages with reaction of 0,5
With the same enthalpy differences between the stator and rotor, the velocity triangles are symmetrical and the shape of the blade passages ($A_1 \neq A_2$) is also symmetrical. $1s_w$ relative stagnation state of working fluid at rotor inlet.

Laval turbine
Pelton turbine
Impulse stage
Reaction stage

Laval turbines[1.] and Pelton turbines show minimal the reaction. Among working machines, low reactions are used in certain fans. At low reaction, compressive force on the rotor is small, calling these stages equal pressure or impulse stages. Conversely, stages with significant reaction yield greater compressive force, termed overpressure or reaction stages.

Axial stage
Flow separation

In axial stages, as reaction increases, blade camber decreases (required momentum change drops), reducing sensitivity to flow separation from the profile decreases.

Equations for calculating velocity distribution at ideal flow in turbomachine

Ideal flow equations are derived for ideal fluid flow without internal losses. Although they are ideal flow equations, they are crucial for basic design of turbomachinery flow components, prediction of properties, analysis of the effect of flow component shape on internal losses of the machine, and understanding the causes of failures or problem operation of turbomachines.

The basic equations describing ideal flow velocities are the potential flow equations. The flow is considered to be potential (meaning that the velocity can only be calculated using the coordinates of a point according to a potential function V=f(x, y, z) in the case of a orthogonal coordinate system, or according to a function V=f(r, θ, a) in the case of a cylindrical coordinate system), where such a flow is referred to as an axisymmetric potential flow. Other quantities of ideal flow can be calculated from the energy equations and the Euler equation of hydrodynamics.

Equation of axisymmetric potential flow

For potential flow, the curl velocity vector must be zero throughout the volume (Equation 8a). For axisymmetric flow, gradients of velocity components in the tangential direction (Equation 8b) must be zero in a cylindrical coordinate system, because the tangential coordinates are closed and the velocity at the origin of the tangential axis must be identical to that at the end of the coordinates. These conditions yield special velocity Equations (8c-h), applicable to other fluid quantities.

$$\left.\begin{array}{ll}\text{(a)} & \mathrm{rot}\quad \vec{V}=0\\ \text{(b)} & \dfrac{1}{r}\dfrac{\partial V_{r,\theta,a}}{\partial \theta}=0\end{array}\right\}\Rightarrow \begin{array}{l}\underset{\text{(c)}}{\dfrac{\partial V_r}{\partial \theta}=0},\ \underset{\text{(d)}}{\dfrac{\partial V_\theta}{\partial \theta}=0},\ \underset{\text{(e)}}{\dfrac{\partial V_a}{\partial \theta}=0},\ \underset{\text{(f)}}{\dfrac{\partial V_\theta}{\partial a}=0},\ \underset{\text{(g)}}{\dfrac{\partial V_r}{\partial a}=\dfrac{\partial V_a}{\partial r}},\\ \underset{\text{(h)}}{\dfrac{1}{r}\dfrac{\partial (r\cdot V_\theta)}{\partial r}=0}\Rightarrow r\cdot V_\theta=C\end{array}$$

8: Axisymmetric potential flow conditions

θ [°] azimuth in the cylindrical coordinate system; C [$m^2 \cdot s^{-1}$] constant (e.g., the proposed magnitude of the product of the tangential component of the absolute velocity V_θ on the mean radius). The modification of the equations is shown in Appendix 16.

Circulation of velocity

Euler work

The product $r\cdot V_\theta$ is called the circulation of the tangential component of the velocity, which is constitutive, so it has the same properties as the irrotational vortex [Škorpík, 2023, p. 1.40]. If the circulation is constant, then the difference of the circulations in front of and behind the rotor is also constant and then also according to the Euler work equation (Equation 4) the Euler work of the potential flow will be constant along the length of the blades, see Problem 8.

Spiral path

The equations of axisymmetric potential flow can also be applied to spiral paths, for example in spiral passages (Problem 9) or in bladeless diffusers and confusers (Problem 10).

Euler equation of hydrodynamics

Pressure gradinet

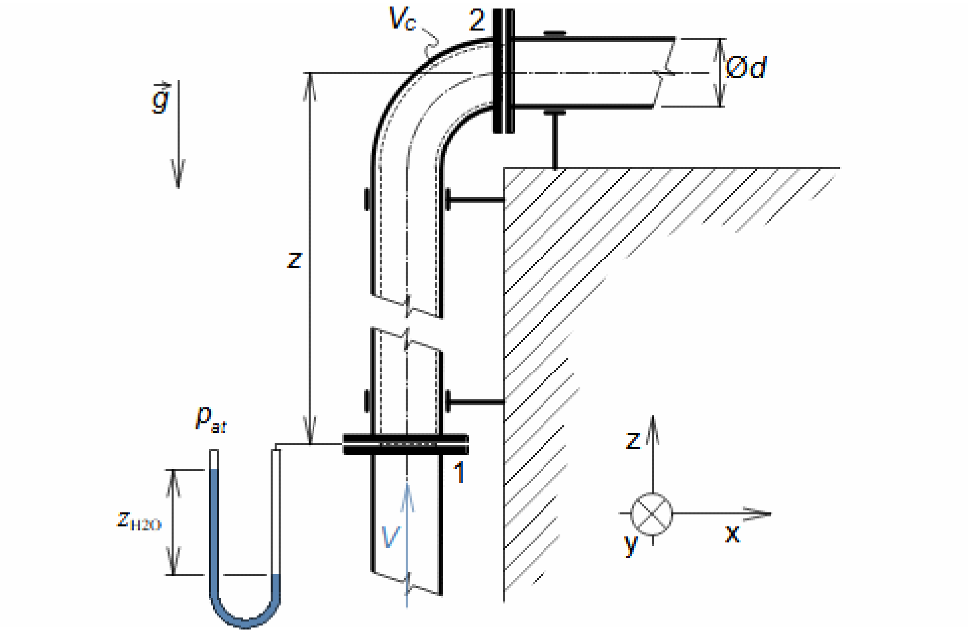
The Euler equation of hydrodynamics for potential flow can also be used to calculate other state variables, for example, the article Internal fluid friction and boundary layer development [Škorpík, 2023b]. From this equation one can read, among other things, that the pressure gradient of a potential flow without the effect of gravitational acceleration cannot have a tangential component, because the gradient of velocity or kinetic energy does not have one either, see Problem 8.

Problems

Problem 1:

Pipes

What force acts on the pipe between flanges due to fluid flow (see figure)? The inner pipe diameter is 23 mm, the flange height difference is 1.2 m, static pressure in the pipe versus outside pressure is 2 m water column, flow velocity is 4 m·s^{-1}, and it's frictionless flow. Water is flowing in the pipe. You are considering frictionless flow. The solution to the problem is shown in Appendix 1.



d [m] pipe diameter; p_{at} [Pa] atmospheric pressure; z [m] altitude coordinate.

1:	entry:	d; z; z_{H2O}; V	**4:**	calculation:	$F_{h,x}$; A; m; p_1; p_2; $F_{p,x}$; F_x
2:	derivation:	eq. for F_x; F_y; F_z	**5:**	calculation:	V_C; m; $F_{h,z}$; F_{pz}; F_z
3:	read off:	ρ; g; p_{at}	**6:**	calculation:	F_y

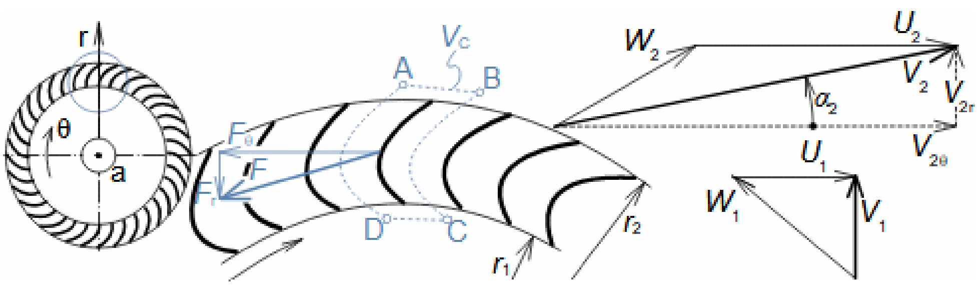
The procedure for solving Problem 1, symbol descriptions are in Appendix 1.

Problem 2:

Force on blade

Radial fan

Determine the force and its components from the fluid flow acting on the radial fan blade. 88,8 kg·h^{-1} of air flows through the fan, the pressure p_1 upstream of the impeller is atmospheric, the pressure difference between the impeller inlet and outlet is insignificant and the number of blades is 52. The other parameters are: r_1=32,5 mm, r_2=37,5 mm, V_1=3,4 m·s^{-1}, V_2=9,34 m·s^{-1}, α_2=18,4°. The width of the impeller is 30 mm. The solution to the problem is shown in Appendix 2.



r [m] r [m] radius; α [°] angle of absolute velocity.

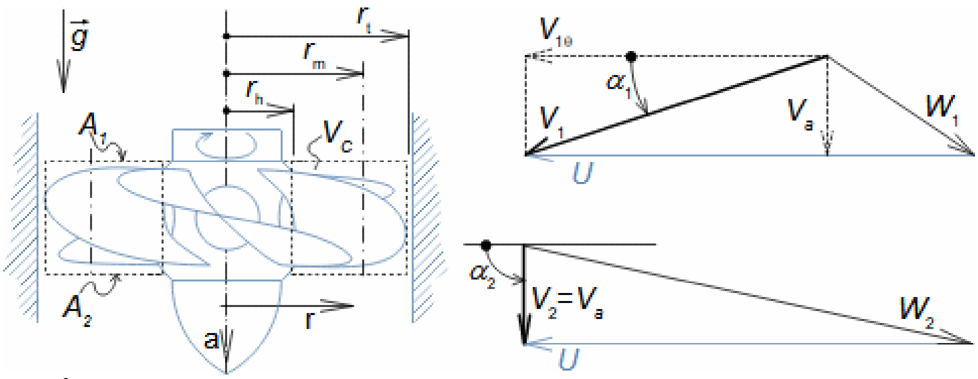
1:	entry:	m; p_1; z; r_1; r_2; $V1$; V_2; α_2; b	**2:**	calculation:	V_{2r}; $F_{r,p}$; F_r
3:	calculation:	$V_{2\theta}$; F_θ	**4:**	calculation:	F

The procedure for solving Problem 2, symbol descriptions are in Appendix 2.

Force on blade
Kaplan turbine

Problem 3:

Calculate force on Kaplan turbine rotor blades from water flow and outlet pressure p_2. Blade tips radius: 1850 mm, hub radius: 985 mm, tangential velocity: 15,3 m·s^{-1}, axial velocity: 13 m·s^{-1}, turbine RPM: 230,8 min^{-1}. 56 m water column above turbine. Velocity triangle shapes at mean radius shown in attached figure. The solution to the problem is shown in Appendix 3.



A [m^2] flow area. The index $_h$ indicates hub of the blade, the index $_m$ indicates the mean square radius of the blade, the index $_t$ indicates the tip of the blade.

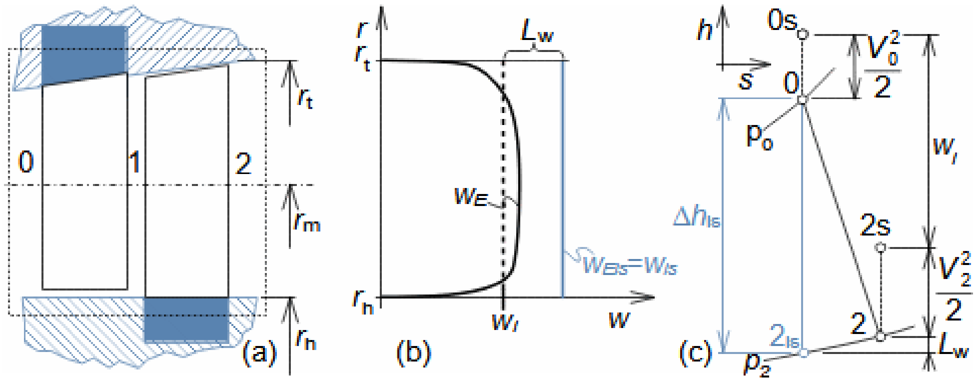
1:	entry:	r_t; r_h; $V_{1\theta}$; V_a; V_2; N; z	**4:**	calculation:	r_m; U; $-W_{2\theta}$; $W_{1\theta}$; $W_{m\theta}$; W_m; β_m; ε; F_a; F
2:	read off:	ρ	**5:**	read off:	p_{at}; g
3:	calculation:	A_1; A_2; Q; m; F_θ	**6:**	calculation:	V_1; p_1; p_2

The procedure for solving Problem 3, symbol descriptions are in Appendix 3.

Euler work
Internal work
Efficiency
1D design

Problem 4:

Calculate the Euler work and Euler efficiency at the mean radius of the axial stage of the steam turbine and the internal work and efficiency of this stage. The stage has been designed by 1D design[1]. hence it has straight blades. The meridional velocity is constant ($V_{0a}=V_{2a}$). The isentropic gradient of the stage is 21,3 kJ·kg^{-1}. The calculated total loss of the stage is 6 kJ·kg^{-1}. The parameters of the velocity triangles at the mean radius are: $V_1=W_2$=148,68 m·s^{-1}, $V_0=V_2=W_1$=63,249 m·s^{-1}, $U_1=U_2$=102,1 m·s^{-1}. The solution of the problem is shown in Appendix 4.



(a) cross-section of stage; (b) Euler work along height of blades; (c) energy balance of stage in *h*-*s* diagram. *h* [J·kg^{-1}] enthalpy; *s* [J·kg^{-1}·K^{-1}] entropy; w_{is} [J·kg^{-1}] internal work of the stage at isentropic expansion (expansion without losses);w_{Eis} [J·kg^{-1}] Euler work at flow without losses; L_w [J·kg^{-1}] internal losses of the stage. The index $_s$ denotes the total state.

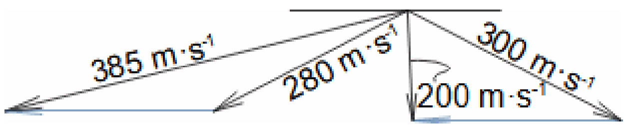
1: entry:	Δh_{is}; L_w; V_1; W_2; V_0; V_2; W_1; U_1; U_2	**3:** calculation:	w_i
2: calculation:	w_E	**4:** calculation:	w_{is}; η_E; η_i

The procedure for solving Problem 4, symbol descriptions are in Appendix 4.

Problem 5:

Reaction

Calculate the reaction of the axial stage of a steam turbine. If you know the velocity triangle. The solution to the problem is shown in Appendix 5.



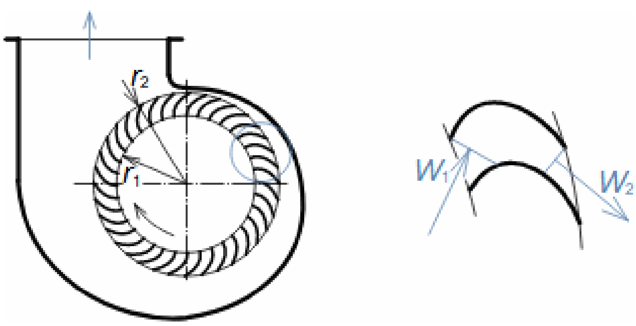
1: entry:	V_1; V_2; W_1; W_2	**3:** calculation:	w_E; Δh_s; R
2: calculation:	Δh_R		

The procedure for solving Problem 5, symbol descriptions are in Appendix 5.

Problem 6:

Reaction
Radial fan

Determine the reaction of a radial fan with forward curved blades if the stagnation pressure increase in the fan is 135 Pa, the working gas density is 1,2 kg·m^{-3}, the blade speed at the outlet of the rotor is 10 m·s^{-1} and the blade speed at the inlet of the rotor is 8,7 m·s^{-1}. The rotor blade passages are designed for equality of relative velocities (W_1=W_2). The solution to the problem is shown in Appendix 6.



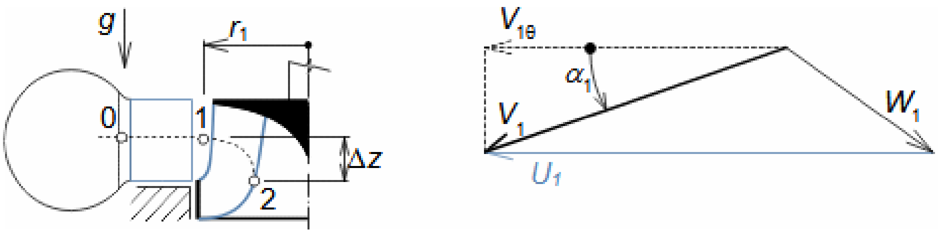
1:	entry:	Δp_s; ρ; U_2; U_1	3:	calculation:	Δh_R; R
2:	calculation:	Δh_s			

The procedure for solving Problem 6, symbol descriptions are in Appendix 6.

Problem 7:

Reaction
Francis turbine

Calculate the reaction of the Francis turbine at its mean radius. The radius of the impeller at the inlet is 1 m. The absolute velocity in front of the impeller is 35 m·s^{-1}, behind the impeller is 12 m·s^{-1} (it has no tangential component). The turbine RPM is 375 min^{-1}. The angle of absolute velocity is 20°. The height difference between the inlet and outlet of the impeller is 0,8 m. The solution to the problem is shown in Appendix 7.



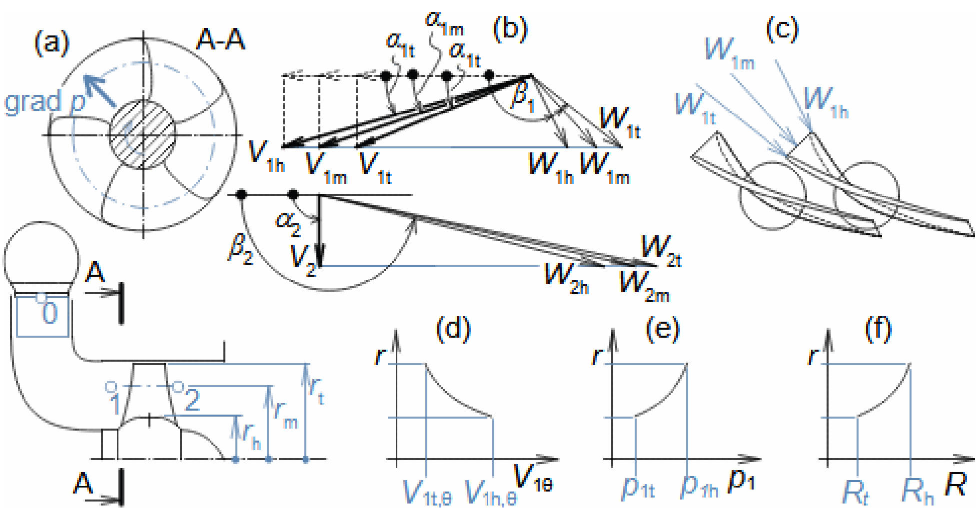
1:	entry:	r_1; V_1; V_2; N; α_1; Δz	4:	derivation:	equation for Δp_R
2:	derivation:	equation for Δp_s	5:	read off:	g; $L_{w,0\text{-}2}$; $L_{w,1\text{-}2}$
3:	calculation:	U_1; $V_{1\theta}$; w_E	6:	calculation:	R

The procedure for solving Problem 7, symbol descriptions are in Appendix 7.

Problem 8:

Velocity triangle
Reaction
Kaplan turbine
Pressure gradient
Twisted blade

Calculate the parameters of the velocity triangle, pressure and reaction at the hub, mean square radius and tip of the Kaplan turbine blade. The required Euler work is 548 J·kg^{-1}. The rotor dimensions, RPM, axial velocity at the mean square radius, and pressure behind the rotor are the same as in Problem 3. The absolute velocity at the rotor exit has only the axial direction. Also determine the pressure gradient in front of and behind the turbine rotor. Consider the potential flow of an ideal fluid. The solution to the problem is shown in Appendix 8.



(a) pressure gradient in front of rotor; (b) changes in absolute and relative velocities at investigated radii; (c) effect of change in relative velocities on shape of blade passage, or blade twist; (d) distribution of tangential component of absolute velocity in front of rotor; (e) distribution of pressure in front of rotor; (f) distribution of reaction along blade height. β [°] angle of relative velocity.

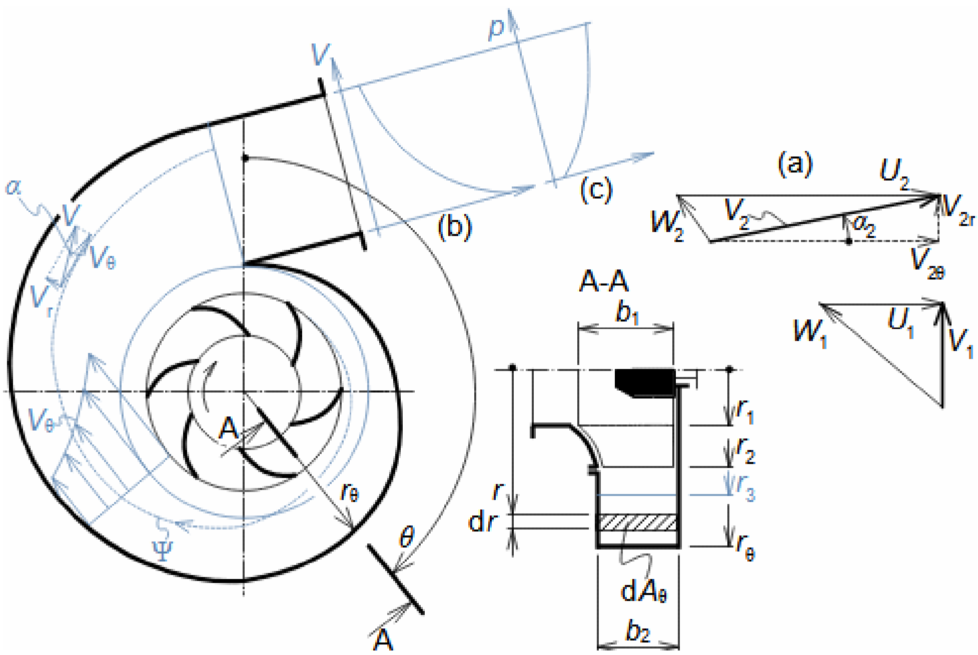
1:	entry:	w_E; r_t; r_h; N; V_a; V_2; p_2; ρ	**6:**	calculation:	p_{1h}; p_{1m}; p_{1t}
2:	calculation:	r_m; U_h; U_m; U_t; $V_{1\theta h}$; $V_{1\theta m}$; $V_{1\theta t}$	**7:**	calculation:	Δp_s; Δp_{Rh}; Δp_{Rm}; Δp_{Rt}; R_h; R_m; R_t
3:	calculation:	V_{1h}; V_{1m}; V_{1t}; α_{1h}; α_{1m}; α_{1t}	**8:**	derivation:	equation for grad p_1
4:	calculation:	$W_{1\theta h}$; $W_{1\theta m}$; $W_{1\theta t}$; W_{1h}; W_{1m}; W_{1t}; β_{1h}; β_{1m}; β_{1t}	**9:**	derivation:	equation for Δp_1
5:	calculation:	W_{2h}; W_{2m}; W_{2t}; β_{2h}; β_{2m}; β2t			

The procedure for solving Problem 8, symbol descriptions are in Appendix 8.

Problem 9:

Spiral casing
Radial fan
Logarithmic spiral

The purpose of the spiral casings of radial machines is to discharge or feed the working fluid from/to the blade section. The flow in such a casing has a spiral path. The figure shows a section of a radial fan with backward curved blades and a spiral casing - suggest the dimensions of this spiral casing if there is a bladeless diffuser between it and the rotor. Determine the pressure at the outlet of the bladeless diffuser (between radii r_2 and r_3). Prove that when an incompressible fluid flows through a radial duct of constant width b, the spiral path is a logarithmic spiral. Discuss the effect of internal friction in the fluid on the shape of the spiral path. What is the velocity and pressure distribution at the exit of the spiral casing? Consider incompressible potential flow. Discuss the effect of casing width on the radius of the casing. The parameters of the fan are R=0,65; r_3=215 mm; r_2=170 mm; r_1=118,5 mm; b_2=101,5 mm; b_1=120 mm; N=1360 min^{-1}. The increase in stagnation pressure in the fan is 500 Pa. The air flow through the fan is 1200 m^3·h^{-1} and its stagnation suction pressure is atmospheric at a density of 1,2 kg·m^{-3}. The solution of the problem and other conclusions are shown in Appendix 9.



(a) velocity triangles; (b) velocity profile at outlet of spiral casing; (c) pressure profile at outlet of spiral casing. Ψ-spiral absolute velocity trajectory.

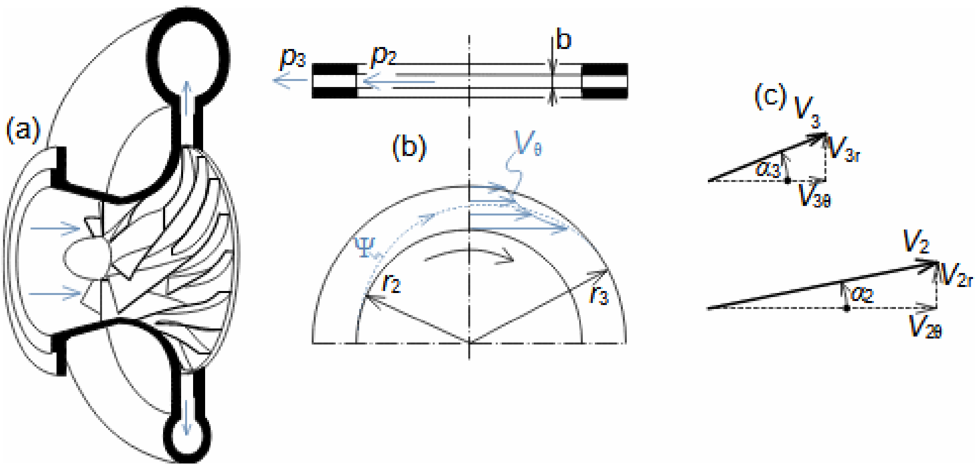
1:	entry:	R; r_3; r_2; r_1; b_2; b_1; Δp_s; N; Q; p_{1s}; ρ	**5:**	calculation:	$p3s$; $V_{3\theta}$; V_{3r}; V_3; p_3
2:	derivation:	equation for r_θ	**6:**	proof:	α=const.
3:	calculation:	w_E; U_2; $V_{2\theta}$; C	**7:**	discussion:	on effect of friction
4:	calculation:	r_θ for selected θ	**8:**	discussion:	velocity and pressure distribution at casing outlet

The procedure for solving Problem 9, symbol descriptions are in Appendix 9.

Problem 10:

bladeless diffuser
Radial compressor

Design the outlet radius of the bladeless diffuser of a radial compressor, which is drawn in the figure. The stage parameters are: $V_{2\theta}$=300 m·s^{-1}, V_{2r}=90 m·s^{-1}, r_2=33 mm, p_2=200 kPa, t_2=82,9 °C. The stator pressure rise is 80 kPa. The working gas is air. Find also whether the angle between the absolute velocity in the bladeless diffuser and its tangential component (between radii r_2 and r_3) changes. Consider the compressible potential flow. The solution to the problem is shown in Appendix 10.



(a) rotor-bladeless diffuser assembly and inlet and outlet casing; (b) bladeless diffuser section; (c) absolute velocity at the inlet and outlet of the bladeless diffuser.

1:	entry:	$V_{2\theta}$; V_{2r}; r_2; p_2; Δp_S; t_2	**4:**	solution:	r_3 z m_2=m_3
2:	read off:	h_3; t_3 from h-s diagram	**5:**	calculation:	α_2; α_3
3:	calculation:	V_3; ρ_3	**6:**	comapare:	α_2 vs α_3

The procedure for solving Problem 10, symbol descriptions are in Appendix 10.

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